Due: Monday, January 27, at the beginning of the lab.

**Question 1**

We consider

\[ f : \mathbb{R} \rightarrow \mathbb{R} \]

\[ x \mapsto \cos x - 6x^2. \]

1.1 Calculate the derivative of \( f \), \( \frac{df}{dx}(x) \)

1.2 Integrate \( f \) over \([0, 2]\): calculate \( \int_0^2 f(x) \, dx \)

1.3 What is the slope of the graph of \( f \) at \( x = 2 \)?
1.4 Prove that the equation $\cos x = 6x^2$ admits a solution on the interval $[0, 2]$. If you can’t think of a way to prove that, then a graphical verification will do.

**Question 2**

We define the following matrices and column vectors:

$$ A = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 4 \\ -1 & 2 & -2 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, $$

Compute the following:

- $Ab$
- $B^T Ab$
- $b^T b$
- $bb^T$
Question 3

For each of the following statements, indicate if it is always true (AT), always false (AF), typically false (TF), or true if a condition is verified (CT), and you will get extra points if your give that condition. Of course, the boundary between TF and CT is fuzzy: I would tend to say that $x^2 + 12 = x$ is typically false while $\|x\| + x = 0$ is conditionally true (true when $x \leq 0$), but one could also say that the first equation is conditionally true (true if $x = 2$ or $x = -1$). We will return to the notion of “typical” behavior later this semester.

- If $A$ and $B$ are two matrices such that $AB$ is defined, then $AB = BA$.

- If $A$ and $B$ are two matrices such that $AB$ is defined, then $(AB)^T = A^T B^T$.

- If $A$ and $B$ are two matrices such that $AB$ is defined, then $(AB)^T = B^T A^T$.

- If $A$ and $B$ are two matrices such that $AB$ is defined and nonsingular (its inverse matrix is defined), then $(AB)^{-1} = A^{-1} B^{-1}$.

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- If $A$ is a square matrix, then its determinant is defined and $\det A^T = \det A$.

- If $A$ is a square matrix, then its determinant is defined and $\det A^T = -\det A$. 
- If $A$ is an $m \times n$ matrix and $B$ is an $n \times p$ matrix, then $AB$ is an $m \times p$ matrix.

- If $A$ is an $m \times m$ matrix, then $\forall \lambda \in \mathbb{R} \ det(\lambda A) = \lambda^n det A$.

- If $A$ and $B$ are both $m \times m$ matrices, then $det(A + B) = det A + det B$.

- If $A$ and $B$ are both $m \times m$ matrices, then $det(AB) = det A \cdot det B$. 
Question 4

In this problem you do not need to evaluate terms such as \( \cos 2 \) or \( \pi^3 \). You need to arrive to an expression that you could use in Maple or muPad to plot a graph.

We consider the following function:

\[
g : \mathbb{R} \rightarrow \mathbb{R} \\
x \mapsto \cos x - 6x^2 \sin x.
\]

1. What is the Taylor polynomial for \( g \), of degree 3, constructed at \( x_0 = 0 \)?

2. What is the Taylor polynomial for \( g \), of degree 2, constructed at \( x_1 = \pi/2 \)?
Question 5

1. Explain why a divergent infinite series such as

\[ S = \sum_{i=1}^{\infty} \frac{1}{n} \]

can have a finite sum in floating point arithmetic.

2. At what point will the partial sums cease to change.

3. If you had to compute this sum for a large value of \( n \), say \( n = 10^{10} \), what could you do to get a result as accurate as possible?