The Venn Diagram is used to graphically show set membership. A box is used to represent the Universal set. Circles, or other closed figures, show sets contained within the Universal set. Shaded regions in the Venn diagram are used to show membership in corresponding sets generated from various set operations.

**Set Operations**

For binary operators, we can apply a three step process to model the operator on a Venn Diagram. First, shade each operand with distinct pattern or color. Second, overlay the two operands. Here we see an overlay of the Venn diagram for the set \( A \) with the Venn Diagram of set \( B \).

For the third step, use the definition of each operator to determine the region to keep.

**Union.** We want to keep elements that are in either set. Keep all shaded regions in the overlay.

**Intersection.** We want to keep elements that are in both sets. Keep the regions with only the combined shading.
**Set Difference.** We want to keep elements that are only in the left operand, but not in the right operand. Keep the regions with the shading of the left operand.

\[ A - B \]

**Symmetric Difference.** We want to keep elements that are in either operand, but not in both operands. Keep the regions with the shading of left or right operand, but not with the combined shading.

\[ A \Delta B \]

**Complement.** Complement is a unary operator. We want to keep elements not in the set.

Keep the unshaded regions of the Venn diagram for the operand set.

\[ \overline{A} \]

**Examples**

- Show that \( A \Delta B = (A \cup B) - (A \cap B) = (A - B) \cup (B - A) \).

For set difference, we keep the region in the overlay that is shaded like the left operand.
For union, we keep all shaded areas.
Looking at the three Venn Diagrams, we see that they all have the same shaded regions, thus we have shown the desired result.

- Show that $A - B = A \cap \overline{B}$.

For intersection, we keep the regions with the combined shading.
Looking at the two Venn Diagrams, we see that they all have the same shaded regions, thus we have shown the desired result.

- Use Venn Diagrams to prove DeMorgan’s Laws. We show the results without commentary.
Prove, using Venn Diagrams, DeMorgan’s Law $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

Left Hand Side

Complement of $A \cap B = \overline{A} \cap \overline{B}$
Right Hand Side

\[ \overline{A} \cup \overline{B} = \overline{A \cup B} \]

Prove, using Venn Diagrams, DeMorgan’s Law \( \overline{A \cup B} = \overline{A} \cap \overline{B} \).

Left Hand Side

\[ \overline{A \cup B} = \overline{A} \cap \overline{B} \]

Right Hand Side

\[ \overline{A} \cap \overline{B} = \overline{A \cap B} \]