1. Upon examination of the claims records of 280 policy holders over a period of five years, an insurance company makes an empirical determination of the probability distribution of $X=$ number of claims in five years.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.32</td>
</tr>
<tr>
<td>1</td>
<td>0.29</td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
</tr>
<tr>
<td>3</td>
<td>0.11</td>
</tr>
<tr>
<td>4</td>
<td>0.06</td>
</tr>
<tr>
<td>5</td>
<td>0.01</td>
</tr>
<tr>
<td>6</td>
<td>0.01</td>
</tr>
</tbody>
</table>

(a) Calculate the expected value of $X$. Show workings.

$$E[X] = \mu_X =$$

(b) Calculate the standard deviation of $X$. Show workings.

$$SD[X] = \sigma_X =$$

(9 Marks)

2. According to a crime report, 60% of people convicted of home burglaries will be convicted of another crime within 5 years of their last conviction. Suppose a random sample of 6 individuals is selected from all those who committed a burglary in 1999. Assume the conditions for the binomial setting are reasonably satisfied. The following table gives the probabilities for a binomial random variable with $n=6$ and probability of success $p=0.60$. Use the table provided to answer this question.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$P(X)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.004096</td>
</tr>
<tr>
<td>1</td>
<td>0.036864</td>
</tr>
<tr>
<td>2</td>
<td>0.138240</td>
</tr>
<tr>
<td>3</td>
<td>0.276480</td>
</tr>
<tr>
<td>4</td>
<td>0.311040</td>
</tr>
<tr>
<td>5</td>
<td>0.186624</td>
</tr>
<tr>
<td>6</td>
<td>0.046656</td>
</tr>
</tbody>
</table>
(a) What is the probability that none of the individuals in the sample will be convicted of another crime within 5 years of their last conviction?

Answer:

(b) What is the chance that all the individuals in the sample will be convicted of another crime?

Answer:

(c) What is the probability that more than 4 of the individuals in the sample will be convicted of another crime?

Answer:

(9 Marks)

3. Scores on a certain nationwide college entrance examination follow a normal distribution with a mean of 500 and a standard deviation of 100. Find the probability that a student will score

(a) over 650.

Answer:

(b) less than 459.

Answer:

(c) between 325 and 675.

Answer:

(d) If a school only admits students who score over 680, what proportion of the students pool would be eligible for admission?

Answer:

(e) What limit (score) would you set that makes the top 20% of the students eligible? (Choose closest number in table. Do not interpolate).

Answer:
(f) If 16 examinations are selected at random and their mean is calculated,

i. what is the expected average score? Answer: 

ii. what is the standard deviation for the average score? Answer: 

iii. is the distribution of the average score exactly or approximately normal? Explain.

   The distribution of the average score is (circle one) exactly / approximately normally distributed.

   Because: ____________________________

iv. What is the probability that the average score of the 16 examinations is over 650?

   Answer: ____________

(22 Marks)

4. Patients with chronic kidney failure may be treated by dialysis, using a machine that removes toxic wastes from the blood, a function normally performed by the kidneys. Kidney failure and dialysis can cause other changes, such as retention of phosphorus, that must be corrected by changes in diet. A study of the nutrition of dialysis patients measured the level of phosphorus in the blood of 45 dialysis patients and their average phosphorus level was 5.2 mg/dl.

(a) Assuming blood phosphorus varies according to a normal distribution, with \( \sigma = 0.9 \) mg/dl, find a 99% confidence interval for the mean blood phosphorus level for dialysis patients.

99% CI: 

(b) What is the margin of error in the 99% confidence interval found in part (a)

Margin of error: 

(c) Interpret the confidence interval found in part (a)

Interpretation: _____________________________________________________________
(d) If the level of confidence is changed to 90%, and all the other parameters stay the same, what effect would lowering the level of confidence have in the size of the confidence interval? Explain.

The 90% confidence interval is (circle one) __________ than the 99% confidence interval.

Because:

(e) How many dialysis patients should be sampled to obtain a margin of error of no more than 0.3 in a 99% confidence interval?

Sample size = __________

(10 Marks)

NOTE: Even though in this practice test there are not questions about hypothesis testing and p-values, be sure there will be in your test!!

**FORMULAE**

The following formulae may be useful:

\[ E[X] = \mu_X = \sum x P(x) \]

\[ \forall [X] = \sigma_X^2 = \sum x^2 P(x) - \mu_X^2 \]

\[ E[X] = \mu_X = np \]

\[ \forall [X] = \sigma_X^2 = npq \]

\[ SD(X) = \sigma_X = \sqrt{npq} \]

\[ Z = \frac{X - \mu}{\sigma} \]

\[ \mu_X = \mu \]

\[ \bar{x} = \frac{\sigma}{\sqrt{n}} \]

\[ \bar{x} = \frac{z_{\alpha/2} \sigma}{\sqrt{n}} \]

\[ n = \left[ \frac{z_{\alpha/2} \sigma}{E} \right]^2 \]

\[ Z = \frac{X - \mu_0}{\sigma/\sqrt{n}} \]

\[ t = \frac{X - \mu_0}{s/\sqrt{n}} \]