PROBABILITY CONCEPTS
(Weiss, Chapter 4)

The science of uncertainty is called **probability theory**.

4.1 Probability Basics

Experiment

- Is a process that results in one of a number of possible outcomes.
- The outcome that occurs cannot be predicted with certainty.

Example

Experiment	Outcome
Flip a coin	
Roll a die	
Dow Jones	

Actual outcome can not be determined (predicted) in advance;

but

can

4.2 Events

Sample space

List of all possible outcomes for an experiment.

i.e. Coin: S =

Event

A collection of outcomes for the experiment

i.e. Dow Jones doesn't increase.

→

Definition - Probability

Number of ways
event can occur

↙

Probability of an event =

↖

Total number of
possible outcomes

Meaning of Probability

Probability is

- a generalization of the concept of percentage.
- a numerical measure of how likely it is that an event will occur.

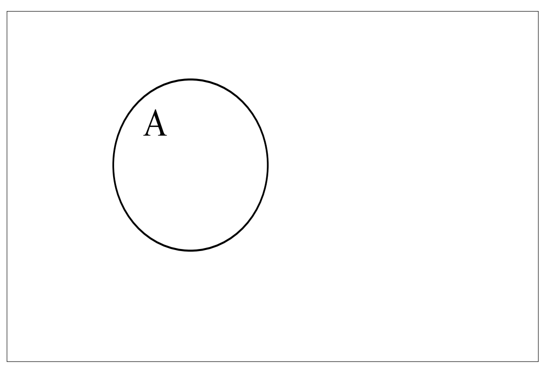
0 0.5 1

Venn Diagrams

Useful for illustrating probability concepts.

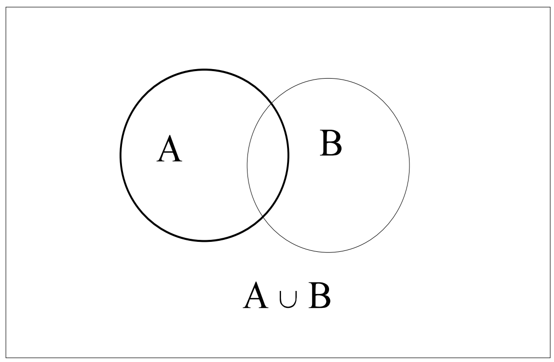
- Sample space - depicted as a rectangle
- Events - disks inside the rectangle

Venn diagram for event A



Union

The **union** of any two events A and B, denoted $A \cup B$ (A **or** B), occurs if



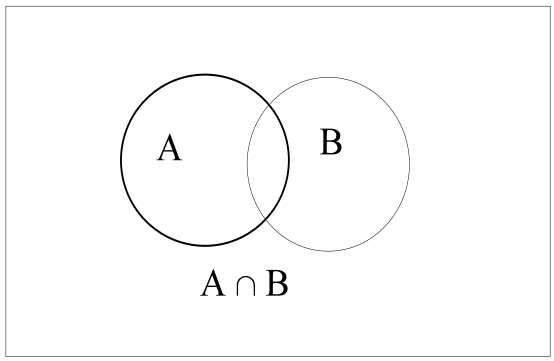
Example - Tossing a die.

Even numbers: C =
Less than of = 3: D =

$C \cup D =$

Intersection

The **intersection** of any two events A and B, denoted A∩B (A&B) occurs



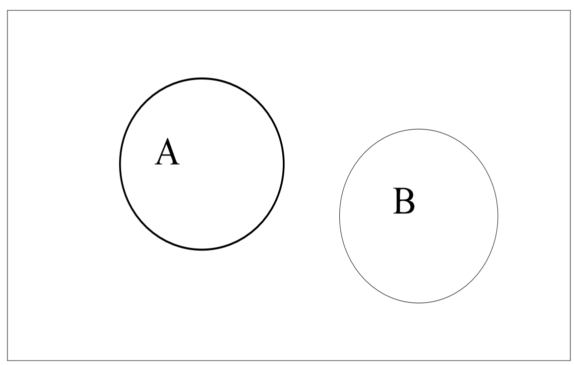
Example

C∩D =

Mutually Exclusive

Two events A and B are mutually exclusive if they do not have outcomes in common.

$$A \cap B =$$



Example

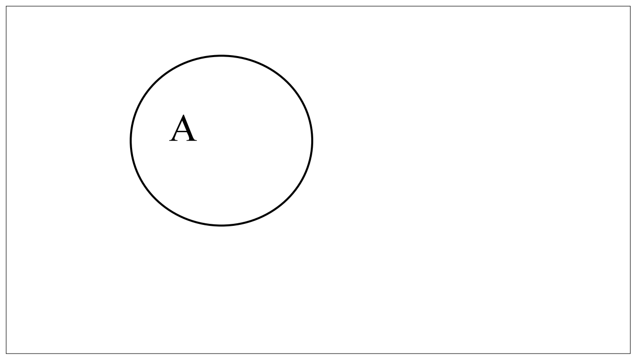
Even numbers: C =
Odd numbers: F =

$$C \cap F =$$

Hence, C & F are

Complement

The **complement** of any event A, denoted $\overline{A}$, is the set of all



Example - $F = \{1, 3, 5\}$

$\overline{F} =$

For any event A,

1. $A \cap \overline{A} =$

2. $A \cup \overline{A} =$

(Exhaustive).

4.3 Some Rules of Probability

1. Probabilities between 0 and 1

$$0 \leq P(A) \leq 1$$

2. Probability of sample space

   . That is:

   P(S) = and P(∅) = P({ }) = .

3. **Addition Rule**

   (Mutually Exclusive Events) :

   When A and B are mutually exclusive

   $P(A \cup B) =$

Ex. Die: A={1} and B={2,4}

$P(A \cup B)$ =

=

= 3/6 = 0.5

Note: can be extended to more than 2 events.

4. **Complement Rule**

$\bar{A}$: Event that A does not occur, then

Know that: $A \cup \bar{A} =$

then $P(A \cup \bar{A}) =$

and

Hence: $P(A) =$

or $P(\bar{A}) =$

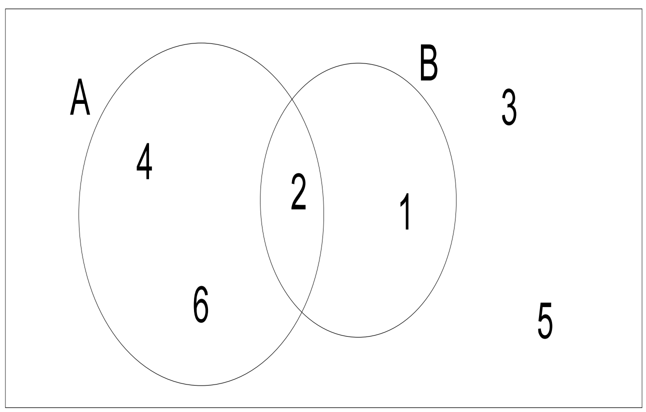
Ex. Die example: Assume that $P(\bar{A}) = 2/6$, what is the P(A)?

P(A) =

5. **Addition Rule (General)** - events A and B do not have to be mutually exclusive events.

$$P(A \cup B) =$$

Ex. A : even number tossed
B : number < 3 tossed



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$=$$

Ex. As reported in *Employment and Earnings*, the age dist'n of employed persons 16 years old and over is

Age	Frequency (000's)	Event
16-19	6,500	A
20-24	12,138	B
25-34	32,077	C
35-44	35,051	D
45-54	25,514	E
55-64	11,739	F
65 & over	3,690	G
Total	126,709	

An employed person is selected at random. Let the following events be defined

W = the person is between 20 and 64
Y = the person is under 65
Z = the person is 55 or over.

Describe each of the following events in words and determine their probabilities.

a. (not Y)

Is the event that the person selected is at least 65 years old.

P (not Y) =

=

=

= = 0.0291

P (Y) = ?

Could use the

P (Y) =

=

b. (not W)

Is the event that the person selected is

$P(\bar{W}) =$

$=$

Since $A \cap G =$

$=$

$=$

$= 0.0804$

c. $Y \cap Z$ (same as)

Is the event that the person selected is

$P(Y \cap Z)$ =

=

= 0.0926

<u>4.4 Joint and Marginal Probabilities</u>

Data obtained by observing values of two variables -on same unit- of a population are called

Frequency distributions of bivariate data is called a

Ex. Following is a contingency table giving the number of institutions of higher education in the US by **region** and **type**.

TYPE / REGION	**Public** T_1	**Private** T_2	**TOTAL**
Northeast R_1	266	555	821
Midwest R_2	359	504	863
South R_3	533	502	1035
West R_4	313	242	555
TOTAL	1471	1803	3274

a. How many cells does this contingency table have?

b. What is the total number of institutions of higher education in the US?

c. How many institutions are in the Midwest?

d. How many are public?

e. How many are private schools in the South?

R_i : Events associated with **Region**
i = 1, 2, 3 & 4

e.i. R_2 : event the higher education institution is in the Midwest

$P(R_2)$ = ?

= = 0.2636

T_i : Events associated with **Type**
i = 1 & 2

e.i. T_1 : event the higher education institution is public

$P(T_1)$ = ?

= = 0.4493

Could also consider events jointly.

e.i. Event the higher education institution is public in the Midwest.

↓

and P( ) = = 0.1097

Note:

$P(R_2)$ and $P(T_1)$ are called

$P(T_1 \cap R_2)$ is called a

4.5 Conditional Probability

The **conditional probability** $P(A|B)$ means the probability that A will occur given that B has occurred.

Ex. Select randomly from standard pack of 52 cards (no replacement). Let

A_1 : 1st card is an ace
A_2 : 2nd card is an ace

$P(A_1) = ?$

$P(A_2|A_1) = ?$

Not always easy to calculate them and need to use **General Formula:**

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \qquad P(B) \neq 0$$

Ex. A joint frequency dist’n for the number of injuries in the US by circumstance and sex is as shown in the following contingency table. Frequencies are in millions.

Circumst. / Sex	**Work** C_1	**Home** C_2	**Other** C_3	**Total**
Male S_1	8.0		17.8	35.6
Female S_2		11.6	12.9	25.8
TOTAL	9.3	21.4	30.7	61.4

a. Fill in the two empty cells

Missing frequencies in cells

b. How many cells does the contingency table have?

c. Find the probability that an injured person was hurt at work.

P( ) = ?

P( ) = = 0.1515

d. Find the probability that the injured person is female.

P(   ) = ?

P(   ) = $=$ 0.4202

e. Find the probability that the injured person is female and was hurt at work.

P(   ) = ?

P(   ) = $=$ 0.0212

f. Given that an individual was hurt at work, what is the probability that it is a female. Obtain this probability directly from the table.

P( ) = ?

P( ) = = 0.1398

g. Obtain $P(S_2|C_1)$ using the conditional probability rule and your answers from part (c) and (e).

$P(S_2|C_1)$ =

$$= \frac{1.3}{9.3} = 0.1398$$

4.6 The Multiplication Rule; Independence

To find the probability of a joint event i.e. $P(A \cap B)$, for any two events A and B, rearrange the conditional probability rule

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

Multiplying both sides by P(B) gives

$P(A \cap B) =$

Alternatively:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}.$$

→

$P(A \cap B) =$

Independent Events

Two events A and B are said to be independent if

$$\boxed{P(A \mid B) = P(A) \quad \text{or} \quad P(B \mid A) = P(B)}.$$

Otherwise the events are dependent.

Multiplication Rule for Independent Events

With independent events $P(A \mid B) = P(A)$ hence, the multiplication rule then reduces to

$$\boxed{P(A \cap B) = \qquad\qquad\qquad}.$$

h. Are S_2 and C_1 independent? Explain.

$$P(S_2 | C_1) \overset{?}{=}$$

$$\overset{?}{=}$$

0.1398 0.4202

Hence, C_1 and S_2 are

i. Obtain $P(S_2 \cap C_1)$ using the multiplication rule and your answers from parts (c) and (f).

$P(S_2 \cap C_1) =$

$$= \quad \times \quad = 0.0212$$