DISCRETE RANDOM VARIABLES
(Weiss, Chapter 5)

5.1 Discrete Random Variable (RV) and Probability Distributions

Random Variable (RV)

Is a variable whose value is determined by an outcome of an experiment

eg. X : Number of defective items in a batch of 50 items

x =

Discrete RV

A discrete RV is a RV whose possible values form a finite (or countably infinite) set of numbers.

Probability Distribution

Specifies the probability associated with each possible value the RV can assume.

Ex. A unit trust is selling for $1.55 per unit. Let X be the RV indicating the price after 1 year (rounded). The RV has the following probability dist'n:

Price after a year (x)	P(x)
1.40	0.10
1.50	0.10
1.60	0.15
1.70	0.25
1.80	0.25
1.90	0.10
2.00	0.05

5.2 The Mean and Standard Deviation of a Discrete RV

Definitions:

i. The **mean of a discrete RV** X is defined by

$\mu_X = \mu =$

Also termed,

ii. The **variance of a discrete RV** X is defined by (shortcut formula)

$$\sigma_X^2 = \sigma^2 =$$

⋮

$\sigma^2 =$

The +ve square root of the **variance** is the **standard deviation** of the RV X.

Ex: Assume the population

1 2 2 2 3 3

What is the mean of the population?

1. Using the full population (data):

$\mu =$

2. Using probability distributions:

x	P(x)	xP(x)
1		
2		
3		

Which one to use?

→ Depends on

Ex. Unit Trust Cont'd

a. What is the expected price of the units after 1 year?

Price after a year (x)	P(x)	xP(x)
1.40	0.10	
1.50	0.10	
1.60	0.15	
1.70	0.25	0.425
1.80	0.25	0.450
1.90	0.10	0.190
2.00	0.05	0.100
	1.00	

↑

→ Hence, the expected price of the unit after 1 year is $1.695.

b. What is the standard deviation in the price of the shares over the 1-year period?

Recall: $\mu=1.695$ & $\sigma^2 = \sum x^2P(x)-\mu^2$

Price after a year (x)	P(x)	x^2	$x^2P(x)$
1.40	0.10		
1.50	0.10		
1.60	0.15		
1.70	0.25	2.89	0.722
1.80	0.25	3.24	0.810
1.90	0.10	3.61	0.361
2.00	0.05	4.00	0.200

$\sigma^2 = \sum x^2P(x) - \mu^2$

$=$

$= 0.025475$

$\rightarrow \sigma =$

5.3 The Binomial Distribution

Characteristic of this dist'n: Only two possible outcomes (Bernoulli trial)

Ex

Conditions

1. Consists of
2. Two outcomes:
3. Trials are
4. Each of the trials has the same probability

→ **Inference on "p : prob. of success" later in chapter 12.**

The **binomial distribution** <u>models chance in repetitions of an experiment with only two outcomes.</u>

The **binomial distribution** with **n** trials and success probability **p** is defined as

$$f(x) = P(X = x) =$$

$\binom{n}{x}$:	# of outcomes that contain exactly **x** successes out of **n** trials
p:	probability of
q:	probability of
x =	

> *Skip calculations. Excel can calculate binomial probabilities for you.*
>
> **=binomdist(x,n,p,false)**

Ex. Suppose 40% of trees in a forest have severe leaf damage from air pollution. If four trees are randomly selected, what are the different probabilities of all the values for the random variable?

Random Variable?

X :

Values for RV X?

x =

Question?

Step 1: Identify a success.

Success :

Step 2: Determine p, the probability of success.

p =
q =

Step 3: Determine n, the number of trials.

n =

Step 4: Binomial probability formula?

Excel : =binomdist(x , n , p , false)

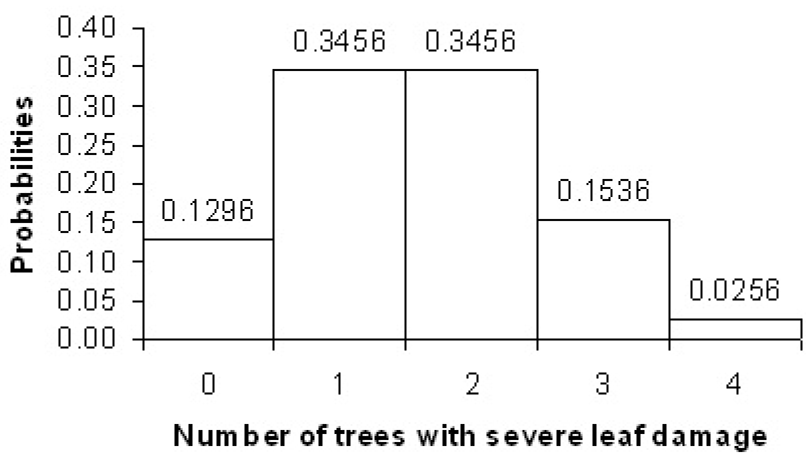
P(X=1) = ?

=

=0.3456

Summary

x	P(X = x)
0	0.1296
1	0.3456
2	0.3456
3	0.1536
4	0.0256



Can answer questions like:

What is the probability of having at least two trees with severe leaf damage?

P(X   2)   =

=

=   0.5248

Equivalently - using the complement rule:

P(X   2)   =

=

=

=

=   0.5248

What is the expected number of trees with severe leaf damage?

$E(X) = ?$ $= \sum xP(x)$

x	P(x)	x P(x)
0	0.1296	
1	0.3456	
2	0.3456	0.6912
3	0.1536	0.4608
4	0.0256	0.1024

$E[X] =$

$= 4 \text{ x } 0.4$

$=$

Following the procedure above to calculate the Variance ($\sigma^2 = \sum x^2 P(x) - \mu^2$), it can be shown that

$$Var(X) = 0.96$$

$$= 4 \times 0.4 \times 0.6$$

$$=$$

Summarizing:

If X is **binomial RV** then

$$\boxed{E[X] = \mu_X = \quad}$$

$$\boxed{Var[X] = \sigma_X^2 = \quad}$$

$$\boxed{SD[X] = \sigma_X = \sqrt{\quad}}$$