

THE SAMPLING DISTRIBUTION OF THE MEAN

Chapter 7

7.1 Sampling Error; The Need for Sampling Dist'ns

Sampling error is the error resulting from using a

Recall:

Descriptive measures of a population are called **parameters**.

Example:

Descriptive measures calculated from a sample are called **statistics**.

Example:

The distribution of a **statistic** is called the

Questions:

- What is the dist'n of \bar{X} ?
- What is the dist'n of \hat{p} ?

7.3 The Sampling Distribution of the sample Mean

Idea: From repeated sampling

Sample 1 of size n →

Sample 2 of size n →

⋮ ⋮

Sample m of size n →

↑

Questions: What is the ?
 What is the ?
 What is the ?

Central Limit Theorem (CLT)

If **relatively large samples** of size n are drawn from **any** population, the sampling dist'n of \bar{X} is approximately normal.

- If the popul'n dist'n **is normal**, the sampling dist'n of \bar{X} will be
- If the population dist'n is **non-normal**, the sampling dist'n of \bar{X} will be,

Mean and SD of \bar{X} ?

7.2 The Mean and SD of \bar{X}

Formula 7.1

The **mean of \bar{X}** , for samples of size n , is equal to the mean of the original popul'n. That is:

$$\mu_{\bar{x}} =$$

Formula 7.2

The **SD for \bar{X}** , for samples of size n , equals the SD of the parent popul'n divided by the square root of the sample size.

$$\sigma_{\bar{x}} =$$

The SD of a statistic is called the **standard error** of the statistic.

$\sigma_{\bar{x}}$: is called the

Example 1

The amount of sulfur in the daily emissions from a power plant has a normal dist'n with a mean of 94 pounds and a SD of 22 pounds.

- a. What is the mean and SD of the parent popul'n?

$X :$

$\mu =$

$\sigma =$

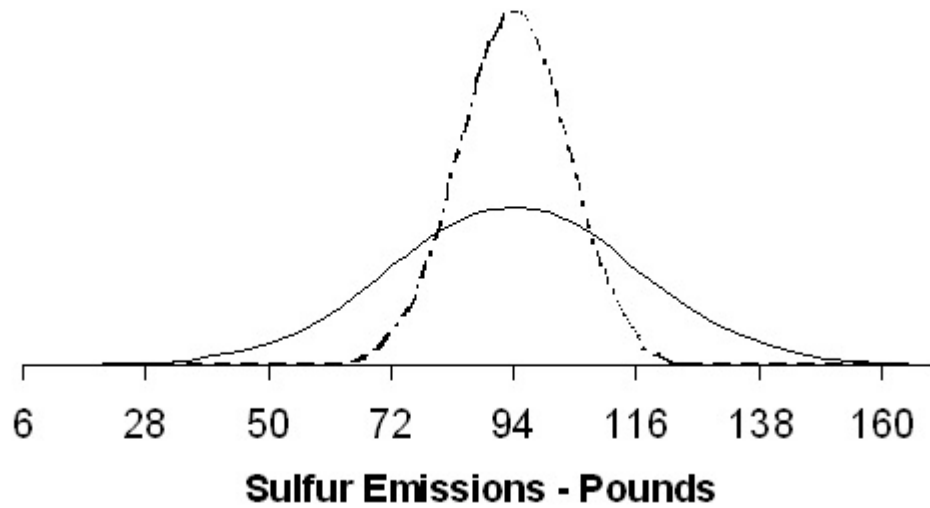
- b. If 5 days are randomly selected and the average sulfur emission is calculated, what is the mean and SD of the sample mean?

Mean: $\mu_{\bar{X}} =$

SD: $\sigma_{\bar{X}} =$

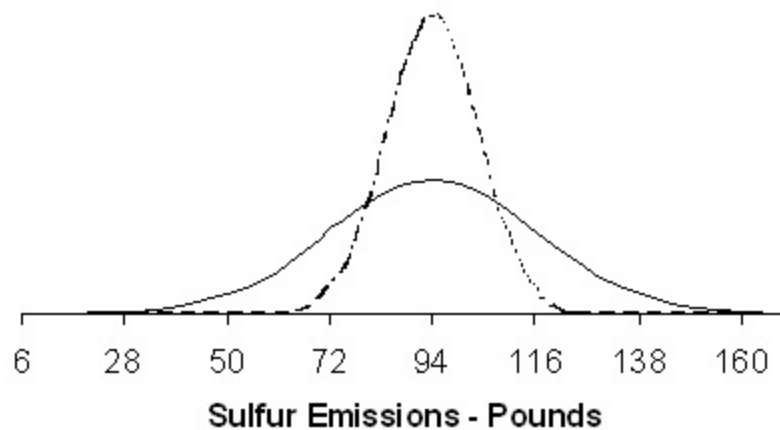
$$\bar{X} \sim N(\quad)$$

- c. Plot the two distributions found in (a) original population and (b) average of 5 observations.



- d. Find the probability that on a randomly chosen day, sulfur emissions are more than 100 pounds.

$$P[\quad 100] = ?$$



$$P [\quad > 100] = P \left[Z > \quad \right]$$

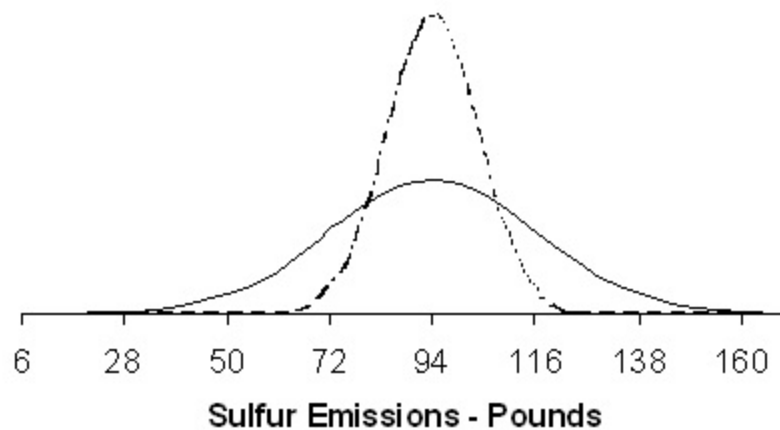
$$= P [Z > \quad]$$

=

=

- e. Find the probability that, if five days are randomly selected, their mean emission exceeds 100 pounds.

$$P[\bar{X} > 100] = ?$$



In this case, the transformation needed to standardized the normal RV is:

$$Z =$$

$$P[X > 100] = P\left[Z > \right]$$

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Example 2

The mean and SD of the strength of a packaging material are 55 and 7 pounds, respectively. If 45 specimens of this material are tested,

- a. Is it reasonable to assume a normal dist'n for the sample mean \bar{X} ? Why or why not?

$$\bar{X} \sim (\quad , \quad)$$

- b. Find the probability that the sample mean strength \bar{X} will be between 54 and 56 pounds?

$$P[54 \leq \bar{X} \leq 56] = ?$$

$$= P \left[\quad < Z < \quad \right]$$

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