Chapter 1 Glossary

alphabet: the set of symbols in a particular formal logic system: constants, variables, functors, and predicates. The alphabet also includes symbols for logical connectives, quantifiers, and auxiliary characters.

application: to apply a substitution (θ) to a variable, term or formula (E) simultaneously replaces in E every free occurrence of each variable (X_i) found in θ with the associated term τ. Variables not in θ are unaffected. See instance.

arity: the number of terms joined in a functor or predicate. The arity can be notated by appending a slash and the arity to the symbol: f/n. E.g., the relation expressed in the natural language statement, “Mary loves Tom” can be formalized as loves(mary, tom) where loves is a predicate of arity 2 (loves/2). Used primarily in writing; not required in Prolog syntax.

atom: a formula with no logical connectives (nor quantifiers); they are the simplest well-formed formulas. Also known as atomic formulas.

auxiliary characters: such as parentheses (“(“ and “)”) and the comma (”,“) used to clarify the formal syntax of logic language. For example, parentheses modify binding and commas separate terms.

bound: a variable X that occurs directly after a quantifier or inside the subformula which follows directly after ∀ X or ∃ X is bound. If not bound, a variable is free.

closed: a formula with no free variables

complete: a set of inference rules is complete if for every set of closed formulas P and every closed formula F, whenever P ⊢ F (F is a logical consequence of P) it holds that P ⊢ F (F is derivable from P). See soundness.

compound term: a composite object constructed with functors

conclusion: a formula that results from applying inference rules to premises

constants: symbols that denote individuals; in Prolog, constants are alphanumeric beginning with lower-case letters. (Numbers are constants, too.)

declarative: in natural language, relating to statements of fact (regardless of their truth). “Mary loves Tom” is a declarative statement – even if she actually despises him.

denote: to identify unambiguously; to associate a literal (in-the-world) meaning with a signifier (symbol). Used to express, e.g., that the symbol t o m refers to a specific individual named Tom.

derive: to draw a conclusion from premises indirectly, using rules of inference

domain: the collection of constants, predicates, and functors that are the objects of a particular logic program. Also known as the universe. Notation: D or |U|

element: some entity in the domain; represented by constants

existential closure: a formula F of the form ∃X1(...(∃XnF)...) where X1,..,Xn are free variables in F. Notation: ∃F

existential quantifier: asserts that at least one such individual exists. Notation: ∃

first-order logic: a formal logical system that uses quantifiers and includes a domain. Predicates are associated with (simple) sets. First-order logic does not allow predicates as arguments to predicates, quantification of predicates, or sets of sets – these appear in higher-order logic systems.

formula: finite sequences of symbols. See also well-formed formulas
**free**: a *variable* which is not *bound*; a place in an expression where substitution (i.e., *application*) may occur.

**functor**: a function over the object domain used to create composite objects. In Prolog, their symbols are alphanumeric, begin with a lower-case letter, and have arity > 0. Note that foo/2 and foo/3 are completely distinct.

**ground**: A *term* or *formula* that contains no *variables* is a ground term (or formula).

**idempotent**: generally, an operation that can be applied any number of times without changing the result. In logic, “a substitution θ is said to be idempotent iff θ = θθ.”

**inconsistent**: a set of *formulas* P is inconsistent if both (P ⊨ F) and (P ⊨ ¬F) can be derived. See *unsatisfiable*.

**individuals**: unique entities in the *world* (not necessarily human beings); denoted by *constants*.

**inference [rules]**: inference rules are formal re-write rules (reasoning principles) that can be used to derive new *formulas* from given ones.

**instance**: a result of applying a substitution to some term or formula. E.g., Eθ is an instance that results from applying θ to E.

**interpretation**: a *structure* plus a mapping from the *symbols* of an alphabet to that structure. (*Constants* map to *elements*; *functors* to *functions*; *predicates* to *relations*.) An interpretation provides a basis for assigning truth values to *formulas*. Notation: _STS_ (black-letter/Fraktur capital “I”).

**logic language**: a programming (or notation) system (such as Prolog) used to perform (or express) calculations within a formal logical system.

**logical connectives**: *symbols* used to combine *formulas* or expressions.

**model**: an *interpretation* is a model of a set of closed formulas iff every one of the formulas is true in the interpretation.

**Modus ponens**: the *inference rule* that whenever the *premises* include both formulas F and F ⊃ G, G can be inferred.

**predicate logic**: Generally, any logical system wherein *variables* can be quantified, but especially *first-order logic*. The kind of logic used in Nilsson and in Prolog.

**predicate**: symbols that denote relations, written as a lowercase word followed by parentheses in which the related terms are listed. E.g., the relation expressed in the natural language statement, “Mary loves Tom” can be formalized as _loves_ (mary, tom) where _loves_ is a predicate of arity 2 (_loves_/2).

**premise**: a *formula* that is given as part of the necessary condition(s) – the “if” clause – of an inference.

**proof**: a sequence of *formulas* where each formula in the sequence is either a *premise* or is derived according to *inference rules*.

**quantified, quantifier, quantification**: see *universal quantifier* and *existential quantifier*.

**relation**: the manner in which *individuals* may be associated in the *world*. In logic programming, a relation is represented by a *predicate*.

**satisfiable**: a description of a *world* in *formulas* that has at least one *model*. See *unsatisfiable*.

**semantics**: the meaning of a statement in a *logic language* as distinct from its form *(syntax)*.

**sound, soundness**: a set of inference rules is sound if for every set of closed formulas P and every closed formula F, whenever P ⊨ F (F is derivable from P) it holds that P ⊨ F (F is a logical consequence of P). See *completeness*.
structure: the algebraic abstraction of the world – a set (of terms?) plus the relations and functions that can be applied to them. See also model.

substitution: a set of variable-to-term mappings: $\theta := \{X_1/t_1, ..., X_n/t_n\}$. Notation: $\theta$ (“theta”) or other Greek letter. See application

symbol: the syntactic representation of a constant, variable, functor, or predicate. (In a logic language, a symbol is encoded as a string of one or more printable characters.)

syntax: the precisely defined form of “legal” statements in a logic language.

term: in predicate logic, a string of symbols representing some constant, variable, or functor; analogous to nouns (and noun phrases) in natural language. (Note that terms do not include operators.)

universal closure: a formula $F$ of the form $\forall X_1(...(\forall X_n F)...)$ where $X_1,...,X_n$ are free variables in $F$. Notation: $\forall F$

universal quantifier: asserts that some expression is true for all such individuals

universe: Also “universe of discourse.” See domain. Compare to world.

unsatisfiable: a set of closed formulas which no interpretation can model is unsatisfiable. For example, if it includes $(F \land \neg F)$. See inconsistent.

valuation: a mapping from variables of the alphabet to the domain of an interpretation. Notation: $\phi$ (“phi”) or other Greek letter

variable: a symbol that refers to some unspecified individual, typically notated as a single capital letter. In Prolog, the symbol is alphanumeric, beginning with a capital letter.

well-formed formulas (wff): formulas (expressions) in a logic language that satisfy the rules of its syntax

world: the “reality” being modeled by the logic program, described as a set of closed formulas. Compare to universe