Handling Quantified Statements

- Valid arguments for quantified statements are a sequence of statements. Each statement is either a premise or follows from previous statements by rules of inference which include:
  - Rules of Inference for Propositional Logic
  - Rules of Inference for Quantified Statements
- The rules of inference for quantified statements are introduced in the next several slides.
Universal Instantiation (UI)

\[ \forall x P(x) \]
\[ \therefore P(c) \]
with domain \( U \) and \( c \in U \)

Example:

Our domain consists of all dogs and Fido is a dog.

“All dogs are cuddly.”

“Therefore, Fido is cuddly.”
Universal Generalization (UG)

\[ P(c) \]
\[ \therefore \forall x P(x) \]

with \( c \in U \) any element in domain \( U \)

Used often implicitly in Mathematical Proofs.
Existential Instantiation (EI)

\[
\exists x P(x) \\
\therefore P(c)
\]

with domain \( U \) and some \( c \in U \)

Example:

“There is someone who got an A in the course.”
“Let’s call her \( a \) and say that \( a \) got an A”
Existential Generalization (EG)

\[ \frac{P(c)}{\therefore \exists x P(x)} \]

with \( c \in U \) some element in domain \( U \)

Example:

“Michelle got an A in the class.”
“Therefore, someone got an A in the class.”
Our Socrates Example

\[\forall x[human(x) \rightarrow mortal(x)]\]

\(human(Socrates)\)

\[\therefore mortal(Socrates)\]

Now we show that the above reasoning step is valid, but constructing a valid argument with the same premises and conclusion:

Let \(U\) be the domain of all objects and \(Socrates \in U\),

(1)\(\forall x[human(x) \rightarrow mortal(x)]\) \hspace{1cm} (premise)
(2)\(human(Socrates) \rightarrow mortal(Socrates)\) \hspace{1cm} (universal instantiation from 1)
(3)\(human(Socrates)\) \hspace{1cm} (premise)
(4)\(mortal(Socrates)\) \hspace{1cm} (modus ponens from 2 and 3)
Universal Modus Ponens

Universal Modus Ponens combines universal instantiation and modus ponens into one rule.

\[ \forall x (P(x) \rightarrow Q(x)) \]

\[ P(a), \text{where } a \text{ is a particular element in the domain} \]

\[ \therefore Q(a) \]

This rule could be used in the Socrates example.