Problems

1. Let $g(x, y, z)$ be a primitive recursive function. Show that the following functions are primitive recursive,
   
   (a) $f(x, y) = g(x, y, x)$
   (b) $f(x, y, z, w) = g(x, y, x)$
   (c) $f(x) = g(1, 2, x)$

2. Show that
   
   $\max(x, y) = \begin{cases} x & \text{if } x \geq y \\ y & \text{otherwise} \end{cases}$

   is primitive recursive.

3. Let $g(x) = x^2$ and $h(x, y, z) = x + y + z$ and let $f(x, y)$ be the function defined from $g$ and $h$ by primitive recursion. Compute the values $f(1, 0), f(1, 1), f(1, 2)$, and $f(5, 0), f(5, 1), f(5, 2)$.

4. The functions below were defined in Table 13.1. Explicitly give the functions $g$ and $h$ that make the definitions legal definitions as given in the definition for primitive recursive functions.

   (a) $sg$
   (b) $sub$
   (c) $exp$