CSC544 Spring 2015
Midterm
Take-Home Exam
Version 1.1

due in class Thursday 4/2/15

Note: All work has to be your own, no team work allowed.

Part A, short answers, 40 points.

1. (5 points) Why do we think of decidable language as representing algorithms?
2. (5 points) What do we mean when we say that two machines are equivalent?
3. (5 points) In your own words; describe the *halting problem*.
4. (5 points) In your own words describe the Church-Turing thesis.
5. (5 points) Compute the normal form of the following λ-expression,

$$(\lambda x.\, \lambda y.\, x\, y)\,(\lambda x.\, x + 1)\, 3$$

6. (5 points) Show that the language

$$L = \{w \mid \text{w has an even number of 0s, or 1s, or both}\}$$

is a regular language.

7. (5 points) What do we mean when we say that *the language L is not Turing-recognizable*?
8. (5 points) What does M accepts the string w mean when M is a nondeterministic automaton or Turing machine?

Part B, problems, 60 points.

1. (10 points) Given the language
$$L = \{ww^R \mid w \in \{a, b\}^*\}$$
where w^R is the reverse of the string w, do the following,
   1. Show that the language L is not regular.
   2. Use a context-free grammar to show the the language L is context-free.

2. (10 points) Let L be the following language
$$L = \{a^n b^n (ab)^n \mid n \geq 0\}$$
and consider the following λ-expression
$$\begin{aligned}
\mathrm{ITER} &= \lambda x.\,(\mathrm{STEP}\,0\,[])\\
\mathrm{STEP} &= \lambda nk.\,(\mathrm{STEP}\,(n+1)\,((\mathrm{GEN}\,n) :: k))\\
\\
\mathrm{GEN} &= \lambda n.\,\mathrm{APPEND}\,(\mathrm{APPEND}\,(\mathrm{ASTRING}\,n)\,(\mathrm{BSTRING}\,n))\,(\mathrm{ABSTRING}\,n)\\
\mathrm{APPEND} &= \lambda xy.\,(x = [])?\,y : (\mathrm{HD}\,x) :: (\mathrm{APPEND}\,(\mathrm{TL}\,x)\,y)\\
\mathrm{HD} &= \lambda x :: y.\,x\\
\mathrm{TL} &= \lambda x :: y.\,y\\
\mathrm{ASTRING} &= \lambda n.\,(n = 0)?\,[] : (a :: (\mathrm{ASTRING}\,(n-1))\\
\mathrm{BSTRING} &= \lambda n.\,(n = 0)?\,[] : (b :: (\mathrm{BSTRING}\,(n-1))\\
\mathrm{ABSTRING} &= \lambda n.\,(n = 0)?\,[] : (a :: b :: (\mathrm{ABSTRING}\,(n-1))
\end{aligned}$$
Prove that the λ-expression ITER generates precisely the set of strings in L. You can assume that the λ-expressoin GEN is correct. You can ignore the fact that ITER generates strings and sequences separates by the :: operator.

3. (10 points) Show that a single tape Turing machine with multiple heads precisely recognizes the Turing-recognizable languages.

4. (10 points) Show that the language (the language of brackets and parentheses)
$$L = \{[^k(^m)^m]^k \mid k \geq 0 \land k \geq m \geq 0\}$$
is recursively enumerable.

5. (20 points) Consider the problem of determining whether a Turing machine M on an input w ever attempts to move its head left when its head is on the left-most tape cell. Let

   $L = \{\langle M, w\rangle \mid M$ on w tries to move its head left from the leftmost tape cell, at some point in its computation$\}$.

   Show that L is undecidable.