Problems

1. Given the grammar

\[
Q \rightarrow Q + D | D \\
D \rightarrow 0 | 1 | DD
\]

Let \(Q_{\text{exp}}\) and \(D_{\text{exp}}\) be the sets defined deductively as follows,

\[
Q_{\text{exp}} = \{ q \mid Q \Rightarrow^* q \land q \in T^* \} \\
D_{\text{exp}} = \{ d \mid D \Rightarrow^* d \land d \in T^* \}
\]

give the inductive definitions of these set.

2. Compute the semantic value of \(ae \equiv 2 \ast v\), where \(ae \in A_{\text{exp}}\) with \(v \in \text{Loc}\) and \(2 \in I\). Assume the initial state \(\sigma_0 \in \Sigma\).

3. Compute the semantic value of \(c \equiv x := 3; \text{if } x \leq 5 \text{ then } x := 0 \text{ else } x := 10 \text{ end}\), where \(c \in \text{Com}\), \(x \in \text{Loc}\), and \(0, 3, 5, 10 \in I\). Assume the initial state \(\sigma_0\).

4. Let \(a_0 \equiv 1 + 1\) and \(a_1 \equiv 2 \ast 2 \ast 2\), where \(a_0, a_1 \in A_{\text{exp}}\). Prove that \(a_0 \sim a_1\).

5. Given the syntax and semantics for the language IMP discussed in class, extend the syntax of this language with the construct \(\text{do } c \text{ while } b \text{ end}\) where \(c \in \text{Com}\) and \(b \in B_{\text{exp}}\). The informal specification of this command is that \(c\) is executed as long as \(b\) remains true. Provide a set of semantic rules that defines the behavior of this construct formally and demonstrate that your rules work with a simple example.

6. Let \(\sigma : \text{Loc} \rightarrow I\) be some state \(\sigma \in \Sigma\), show that \(\sigma[2/x] = (\sigma[1/x])[2/x]\) for some \(x \in \text{Loc}\). (Hint: use extensional equality of functions.\(^1\))

Where not stated explicitly otherwise, show your computations based on the semantic rules covered in class.\(^2\)

\(^1\)http://en.wikipedia.org/wiki/Extensionality
\(^2\)Typewritten work is preferred :)

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