1. ♦ Let $T$ be a tree with more than one node.
   (a) Is it possible that a preorder traversal visits the nodes of $T$ in the same order as a postorder traversal? If so, provide an example, otherwise argue why this cannot occur.
   (b) Is it possible that a preorder traversal visits the nodes of $T$ in the reverse order as a postorder traversal? If so, provide an example, otherwise argue why this cannot occur.

2. ♦ Let $T$ be a full binary tree with $n$ nodes, and let $h$ denote the height of $T$.
   (a) Draw a binary tree of height $h = 4$ and maximum number of external nodes (leaves).
   (b) What is the minimum number of external nodes for $T$? Justify your answer.
   (c) What is the maximum number of external nodes for $T$? Justify your answer.
   (d) What is the minimum number of internal nodes for $T$? Justify your answer.
   (e) What is the maximum number of internal nodes for $T$? Justify your answer.

3. ★ Describe, in pseudocode, a link-hopping method for finding the middle node of a doubly linked list with head and tail pointers, and an odd number of nodes. Note that you should only use link-hopping (cannot use a counter).

4. ★ Give a $O(n)$ time algorithm for computing the depth of all the nodes of a tree, where $n$ is the number of nodes of the tree.

5. ♦ Consider a binary tree $T$. Let the balance factor of an external node $v$ be 0 and the balance factor of an internal node be the difference between the heights of the right and left subtrees of $v$. Describe an algorithm for computing the balance factor of all the nodes of $T$.

6. ♦ Show that the maximum number of nodes in a binary tree of height $h$ is $2^{h+1} - 1$.

7. ★ Suppose that items 10, 20, 30, 40, 50 are enqueued, in that order, onto an initially empty queue $Q$. Then a sequence of four dequeue operations are performed on $Q$; as each item is dequeued, it is inserted into an initially empty stack $S$. If two items are then popped off the stack $S$, what is the item to be returned by the next pop operation?

8. ♦ Suppose you have two objects $P$ and $Q$, from a class Queue with standard member functions: enqueue, dequeue, isempty, and size. How would you implement the two stack operations pop and push? Give the running time of each operation.
9. What is the output of the following code?

```cpp
int values[] = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9};
std::stack<int> mystack;
for (int i = 0 ; i < 10 ; i ++)
    mystack.push(values[i]);
int n = 20;
for (int i = 0 ; i < 5 ; i ++)
    n += mystack.pop();
for (int i = 8 ; i >= 1 ; i /= 2)
    n -= mystack.pop();
std::cout << n;
```

10. Draw the resulting binary search tree after the insertion of: 5, 9, 14, 7, 3, 4, 23, 18, 1, 2, 11, 10, 16, 21.

11. Draw the binary search trees of minimum and maximum heights that store all the integers in the range from 1 to 7, inclusive.

12. Draw the sequence of binary search trees that results after deleting items 21, 23, 14, 1, 5, in this order, from the tree in problem 10.

13. Considering the definition of height of a Binary Search Tree (BST), give a formula for:

   (a) the maximum height of a BST with \( n \) nodes?
   (b) the minimum height of a BST with \( n \) nodes?
   (c) the maximum number of nodes in a BST of height \( h \)?
   (d) the minimum number of nodes in a BST of height \( h \)?

14. Suppose that a binary search tree \( T \) is constructed by inserting integers 1 to \( n \), in this order. Give a big-O characterization of the number of comparisons that were done to construct \( T \).