Stacks and Queues

1. Suppose that items 10, 20, 30, 40, 50 are enqueued, in that order, onto an initially empty queue $Q$. Then a sequence of four dequeue operations are performed on $Q$; as each item is dequeued, it is inserted into an initially empty stack $S$. If two items are then popped off the stack $S$, what is the item to be returned by the next pop operation?

2. Suppose you have two objects $P$ and $Q$, from a class Queue with standard member functions: enqueue, dequeue, isempty, and size. How would you implement the two stack operations pop and push? Give the running time of each operation.

3. What is the output of the following code?

   ```cpp
   int values[] = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9};
   std::stack<int> mystack;
   for (int i = 0 ; i < 10 ; i ++)
       mystack.push(values[i]);
   int n = 20;
   for (int i = 0 ; i < 5 ; i ++)
       n += mystack.pop();
   for (int i = 8 ; i >= 1 ; i /= 2)
       n -= mystack.pop();
   std::cout << n;
   ```

4. Suppose you are given an array $A$ containing $n$ numbers in order. Describe in pseudocode an efficient algorithm for reversing the order of the numbers in $A$ using a single for-loop that indexes through the cells of $A$, to insert each element into a stack, and then another for-loop that removes the elements from the stack and puts them back into $A$ in reverse order. What is the running time of this algorithm?

5. Solve the previous exercise using a queue instead of stack.
Linked Lists

1. Describe, in pseudocode, a link-hopping method for finding the middle node of a doubly linked list with head and tail pointers, and an odd number of nodes. Note that you should only use link-hopping (cannot use a counter).

2. Describe an implementation of void insertBefore(Node *p, int val), for a doubly linked list. The method should insert a new node with the element val before the node pointed by p.

3. Assuming efficient implementation of linked-lists (LL), indicate the worst-case performance of each of the following operations (in terms of n). OBS: Consider that you can only have the pointer head in your implementation.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Singly LL</th>
<th>Doubly LL</th>
<th>Circular SLL</th>
<th>Circular DLL</th>
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</thead>
<tbody>
<tr>
<td>getFirst()</td>
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<tr>
<td>getLast()</td>
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<tr>
<td>prepend(val)</td>
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<td>append(val)</td>
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<tr>
<td>insertAt(index, val)</td>
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<td>removeFirst()</td>
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<td>removeLast()</td>
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<td>removeAt(index)</td>
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<tr>
<td>contains(val)</td>
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</table>

Trees and BSTs

1. Let T be a tree with more than one node.

   (a) Is it possible that a preorder traversal visits the nodes of T in the same order as a postorder traversal? If so, provide an example, otherwise argue why this cannot occur.

   (b) Is it possible that a preorder traversal visits the nodes of T in the reverse order as a postorder traversal? If so, provide an example, otherwise argue why this cannot occur.

2. Let T be a full k-ary tree, where \( k = 2 \) (a.k.a. binary tree), with \( n \) nodes. Let \( h \) denote the height of \( T \).
(a) What is the minimum number of leaves for $T$? Justify your answer.
(b) What is the maximum number of leaves for $T$? Justify your answer.
(c) What is the minimum number of internal nodes for $T$? Justify your answer.
(d) What is the maximum number of internal nodes for $T$? Justify your answer.

3. Give a $O(n)$ time algorithm for computing the depth of all the nodes of a tree, where $n$ is the number of nodes.

4. Show that the maximum number of nodes in a binary tree of height $h$ is $2^{h+1} - 1$.

5. Considering the definition of height of a binary tree, give a formula for:
   (a) the maximum height of a binary tree with $n$ nodes?
   (b) the minimum height of a binary tree with $n$ nodes?
   (c) the maximum number of nodes in a binary tree of height $h$?
   (d) the minimum number of nodes in a binary tree of height $h$?

6. Draw the resulting BST after inserting: 5, 9, 14, 7, 3, 4, 23, 18, 1, 2, 11, 10, 16, 21, 44, 75, 51.

7. Draw the sequence of BSTs that results after deleting items 21, 23, 14, 1, 5, 18, in this order, from the tree in problem 6.

8. Draw BSTs of minimum and maximum heights that store all the integers in the range from 1 to 7, inclusive.

9. Suppose that a BST $T$ is constructed by inserting integers 1 to $n$, in this order. Give a big-O characterization of the number of comparisons that were done to construct $T$.

10. Consider a BST $T$. Let the height of an empty tree be $-1$ and the balance of a node $x$ be the difference between the heights of the left and right subtrees of $x$, as calculated by: $\text{height}(v->\text{left}) - \text{height}(v->\text{right})$. Write an algorithm for computing the balance of each node of $T$. 