Interpretation The possible sample mean IQs for samples of four people have a normal distribution with mean 100 and standard deviation 8.

For samples of size 16, we have $16/\sqrt{n} = 16/\sqrt{16} = 4$, and therefore the sampling distribution of the sample mean is a normal distribution with mean 100 and standard deviation 4. Figure 7.4(c) shows this normal distribution.

Interpretation The possible sample mean IQs for samples of 16 people have a normal distribution with mean 100 and standard deviation 4.

The normal curves in Figs. 7.4(b) and 7.4(c) are drawn to scale so that you can visualize two important things that you already know: both curves are centered at the population mean ($\mu_\bar{x} = \mu$), and the spread decreases as the sample size increases ($\sigma_\bar{x} = \sigma/\sqrt{n}$).

Figure 7.4 also illustrates something else that you already know: The possible sample means cluster more closely around the population mean as the sample size increases, and therefore the larger the sample size, the smaller the sampling error tends to be in estimating a population mean by a sample mean.

**Central Limit Theorem**

According to Key Fact 7.2, if the variable $x$ is normally distributed, so is the variable $\bar{x}$. That key fact also holds approximately if $x$ is not normally distributed, provided only that the sample size is relatively large. This extraordinary fact, one of the most important theorems in statistics, is called the **central limit theorem**.

**KEY FACT 7.3**

**What Does It Mean?**

- For a large sample size, the possible sample means are approximately normally distributed, regardless of the distribution of the variable under consideration.

**The Central Limit Theorem (CLT)**

For a relatively large sample size, the variable $\bar{x}$ is approximately normally distributed, regardless of the distribution of the variable under consideration. The approximation becomes better with increasing sample size.

Roughly speaking, the farther the variable under consideration is from being normally distributed, the larger the sample size must be for a normal distribution to provide an adequate approximation to the distribution of $\bar{x}$. Usually, however, a sample size of 30 or more ($n \geq 30$) is large enough.

The proof of the central limit theorem is difficult, but we can make it plausible by simulation, as shown in the next example.
a. Determine the sampling distribution of the sample mean for simple random samples of 50 women with one job. Explain your reasoning.

b. Obtain the probability that the sampling error made in estimating the mean length of time employed by all women with one job by that of a random sample of 50 such women will be at most 20 weeks.

7.75 Air Conditioning Service Contracts. An air conditioning contractor is preparing to offer service contracts on the brand of compressor used in all of the units her company installs. Before she can work out the details, she must estimate how long those compressors last, on average. The contractor anticipated this need and has kept detailed records on the lifetimes of a random sample of 250 compressors. She plans to use the sample mean lifetime, \( \bar{x} \), of those 250 compressors as her estimate for the population mean lifetime, \( \mu \), of all such compressors. If the lifetimes of this brand of compressor have a standard deviation of 40 months, what is the probability that the contractor's estimate will be within 5 months of the true mean?

7.76 Prices of History Books. The R. R. Bowker Company collects information on the retail prices of books and publishes its findings in The Bowker Annual Library and Book Trade Almanac. In 2005, the mean retail price of all history books was \$78.01. Assume that the standard deviation of this year's retail prices of all history books is \$7.61. If this year's mean retail price of all history books is the same as the 2005 mean, what percentage of all samples of size 40 of this year's history books have mean retail prices of at least \$81.44? State any assumptions that you are making in solving this problem.

7.77 Poverty and Dietary Calcium. Calcium is the most abundant mineral in the human body and has several important functions. Most body calcium is stored in the bones and teeth, where it functions to support their structure. Recommendations for calcium are provided in Dietary Reference Intakes, developed by the Institute of Medicine of the National Academy of Sciences. The recommended adequate intake (RAI) of calcium for adults (ages 19–50) is 1000 milligrams (mg) per day. If adults with incomes below the poverty level have a mean calcium intake equal to the RAI, what percentage of all samples of 18 such adults have mean calcium intakes of at most 947.4 mg? Assume that \( \sigma = 188 \text{ mg} \). State any assumptions that you are making in solving this problem.

7.78 Early-Onset Dementia. Dementia is the loss of the intellectual and social abilities severe enough to interfere with judgment, behavior, and daily functioning. Alzheimer's disease is the most common type of dementia. In the article "Living with Early Onset Dementia: Exploring the Experience and Developing Evidence-Based Guidelines for Practice" (Alzheimer's Care Quarterly, Vol. 5, Issue 2, pp. 111–122), P. Harris and J. Keady explored the experience and struggles of people diagnosed with dementia and their families. If the mean age at diagnosis of all people with early-onset dementia is 55 years, find the probability that a random sample of 21 such people will have a mean age at diagnosis less than 52.5 years. Assume that the population standard deviation is 6.8 years. State any assumptions that you are making in solving this problem.

7.79 Worker Fatigue. A study by M. Chen et al. titled "Heat Stress Evaluation and Worker Fatigue in a Steel Plant" (American Industrial Hygiene Association, Vol. 64, pp. 352–359) assessed fatigue in steel-plant workers due to heat stress. If the mean post-work heart rate for casting workers equals the normal resting heart rate of 72 beats per minute (bpm), find the probability that a random sample of 29 casting workers will have a mean post-work heart rate exceeding 78.3 bpm. Assume that the population standard deviation of post-work heart rates for casting workers is 11.2 bpm. State any assumptions that you are making in solving this problem.

### Extending the Concepts and Skills

Use the 68.26-95.44-99.74 rule (page 271) to answer the questions posed in parts (a)-(c) of Exercises 7.80 and 7.81.

7.80 A variable of a population is normally distributed with mean \( \mu \) and standard deviation \( \sigma \). For samples of size \( n \), fill in the blanks. Justify your answers.

a. \( \mu \) is the mean of __________ of the population mean, \( \mu \).

b. \( \mu \) is the mean of __________ of the population mean, \( \mu \).

c. \( \mu \) is the mean of __________ of the population mean, \( \mu \).

d. \( 100(1 - \alpha) \) of all possible samples have means that lie within __________ of the population mean, \( \mu \). (Hint: Draw a graph for the distribution of \( \bar{x} \), and determine the z-scores dividing the area under the normal curve into a middle \( 1 - \alpha \) area and two outside areas of \( \alpha/2 \).)

7.81 A variable of a population has mean \( \mu \) and standard deviation \( \sigma \). For a large sample size \( n \), fill in the blanks. Justify your answers.

a. Approximately __________ of all possible samples have means within \( \sigma/\sqrt{n} \) of the population mean, \( \mu \).

b. Approximately __________ of all possible samples have means within \( 2\sigma/\sqrt{n} \) of the population mean, \( \mu \).

c. Approximately __________ of all possible samples have means within \( 3\sigma/\sqrt{n} \) of the population mean, \( \mu \).

d. Approximately __________ of all possible samples have means within \( za/2 \) of the population mean, \( \mu \).

7.82 Testing for Content Accuracy. A brand of water-softener salt comes in packages marked "net weight 40 lb." The company that packages the salt claims that the bags contain an average of 40 lb of salt and that the standard deviation of the weights is 1.5 lb. Assume that the weights are normally distributed.

a. Obtain the probability that the weight of one randomly selected bag of water-softener salt will be 39 lb or less, if the company's claim is true.

b. Determine the probability that the mean weight of 10 randomly selected bags of water-softener salt will be 39 lb or less, if the company's claim is true.

c. If you bought one bag of water-softener salt and it weighed 39 lb, would you consider this evidence that the company's claim is incorrect? Explain your answer.

d. If you bought 10 bags of water-softener salt and their mean weight was 39 lb, would you consider this evidence that the company's claim is incorrect? Explain your answer.

7.83 Household Size. In Example 7.9 on page 312, we conducted a simulation to check the plausibility of the central limit theorem. The variable under consideration there is household size, and the population consists of all U.S. households. A frequency distribution for household size of U.S. households is presented in Table 7.7.