Hypothesis Tests for One Population Mean
(Chapter 9)

9.1 & Nature of Hypotheses Testing
9.2 Critical Value Approach to Hypothesis Testing

Objective:

To make decisions or judgments about the value of a parameter (such as \( \mu \)) based on a sample.

The steps in hypothesis testing -for a single population mean \( \mu \)- are:

Example:

Interested in the mean daily caloric intake in the adult rural population of a developing country.
1. We establish a **null hypothesis**

   \[ H_0 : \text{(Null : "No Difference")} \]

2. A nutritionist is interested to prove that it is less than 2000.

   Therefore we establish an **alternative hypothesis** (primary concern):

   \[ H_a : \]
$H_a$ takes one of the three forms:

- $H_a: \mu < \mu_0$  
  e.g.

- $H_a: \mu > \mu_0$  
  e.g.

- $H_a: \mu \neq \mu_0$  
  e.g.

(See examples 9.1 - 9.3)

**Statistical Decision:**

\[
\begin{align*}
\text{Accept } H_0 & \quad \text{or} \quad \text{Reject } H_0 \\
\downarrow \\
(\text{Fail to reject } H_0) \\
\downarrow \\
(\text{Do not reject } H_0)
\end{align*}
\]
3. **Test Statistic**

   Is the criterion upon which we base our decision.

4. **Rejection (Acceptance) Region**

   Is a range of values such that, if the test statistic falls into that range, we decide

   **Critical Values**

   The values that separate the
Type I and Type II Errors

In hypothesis testing it is possible to reach the incorrect decision

<table>
<thead>
<tr>
<th>Statistical Test</th>
<th>The real World</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Fail to reject } H_0 )</td>
<td>( H_0 ) is True</td>
</tr>
<tr>
<td>( \text{Accept } H_0 )</td>
<td>( H_0 ) is false</td>
</tr>
<tr>
<td>( \text{Reject } H_0 )</td>
<td></td>
</tr>
</tbody>
</table>

**Type I error** is the probability of rejecting a true null hypothesis \( H_0 \). (Analogy - Sending an innocent person to jail.)

**Type II error** is the probability of accepting a false \( H_0 \). (Analogy - Freeing a guilty person.)
Significance levels

\[ P(\text{Type I error}) = \]

Would like \( \alpha \) as small as possible - Chosen by the decision maker.

The most common significance levels for \( \alpha \) are:

\[ \alpha = 0.01, \quad 0.05, \quad 0.10 \]

1%  5%  10%
Note:
A one tail test have the rejection region located in one tail:

A two tail test have the rejection region located in both tails:
9.4 Hypothesis Test for One Population Mean ($\mu$) when $\sigma$ is Known.

Assume: normal like popul’n or large sample.

1. State $H_0$ and $H_a$
   
   $H_0 : \mu = $  
   $H_a : \mu < $  
   $> $  
   $\neq$

2. Decide on the significance level, $\alpha$.

3. Find the critical value.

4. Compute the value of the test statistic

   $$Z =$$

5. Draw the conclusion.
Example 1

A study by researchers at the Univ. of Maryland addressed the question of whether the mean body temperature of humans is $98.6^\circ F$. The researchers obtained the body temperatures of 93 healthy humans and the mean is $98.4^\circ F$.

At the 1% significance level, do the data provide sufficient evidence to conclude that the mean body temperature of healthy humans differs from $98.6^\circ F$? Assume $\sigma=0.63^\circ F$.

\[ \mu = \quad n = \]
\[ \bar{x} = \quad \sigma = \]
1. $H_0 : \quad H_a :$

2. $\alpha =$

3.

4. Test statistics:

Do we know $\sigma$?  
Is the sample size large?

Hence, the test statistics to use is:

$$Z = \frac{-0.20}{0.0653} = -3.06$$
5. Reject $H_0$ if

Since

That is, at the 1% significance level, the data provide sufficient evidence to conclude that the mean body temperature of all healthy humans

- What type of error are we likely to commit in this test?

Type
Example 2

A brochure inviting subscriptions for a new diet program states that the participants are expected to lose over 22 pounds in five weeks.

Suppose that from the data of the five-weeks weight losses of 56 participants, the sample mean is found to be 23.5. Could the statement in the brochure be substantiated on the basis of these findings? Test with $\alpha=0.05$ and assume $\sigma=10.2$.

$$\mu = \quad n = \quad$$

$$\bar{x} = \quad \sigma = \quad$$
1. $H_0 : \quad H_a :$

2. $\alpha =$

3. 

4. Test statistics:

Do we know $\sigma$?  
Is the sample size large?

$$Z = \frac{1.5}{1.363} = 1.10$$
5. Reject $H_0$ if

Since

That is, at the 5% significance level, the data provide sufficient evidence to conclude that

- What type of error are we likely to commit in this test?

Type
9.3 P-value Approach to Hypothesis Testing

The p-value of a test of hypothesis is the smallest value of \( \alpha \) that would lead to rejection of the null hypothesis.

**Example** (Example 2 above - Weight loss)

- A one tail test (\( H_a : \mu > 22 \))

  The test statistics \( Z = 1.10 \)

Then the p-value is calculated as:

\[
p\text{-value} = \\
= \\
= \\
= 0.1357
\]
Interpreting the p-value

One tail example : (\(H_a : \mu > 22\))
\(\alpha = .05\), \(Z = 1.10\) and p-value = 0.1357.

Since p-value =
The p-value of a two tail test is computed slightly different:

**Example** (Example 1 above - Body Temp.)

Since the rejection region in a two tail test \( (H_a: \mu \neq 98.6) \) is

\[
Z > z_{\alpha/2} \quad \text{and} \quad Z < -z_{\alpha/2}
\]

the probability that \( Z \) is smaller than the test statistics value (of -3.06) should be doubled.

\[
p-value =
\]

\[
= 0.0022
\]
Interpretation:

Two tail example: ( $H_a: \mu \neq 98.6$ )
$\alpha = .01, Z = -3.06 \text{ and } p\text{-value} = 0.0022$

Since $p\text{-value} =$
Calculation of the p-value

Let $z_a$ be the actual value of the test statistic, and let $\mu_0$ be the hypothesized value:

- If $H_a : \mu > \mu_0$ then

  \[ p\text{-value} = \]

- If $H_a : \mu < \mu_0$ then

  \[ p\text{-value} = \]

- If $H_a : \mu \neq \mu_0$ then

  \[ p\text{-value} = \]
  \[ = \]

(See examples 9.8, 9.9 & 9.10).
In general:

- If p-value > \( \alpha \) then do not reject the null hypothesis \( H_0 \).

- If p-value < \( \alpha \) then reject the null hypothesis \( H_0 \).

* Computer programs will produce p-values for you.

* Advantage: you do not need tables.
9.5 **Hypothesis Test for One Population Mean ($\mu$) when $\sigma$ is Unknown**

Recall:

When $n>30$ or popul’n is normal and $\sigma$ is unknown, the studentized version of $\bar{x}$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

has a t-dist’n with $df =$
The one-sample t-test for $\mu$:

**Assume:** normal popul’n or large sample and $\sigma$ unknown.

1. State $H_0$ and $H_a$
   
   $H_0 : \mu = \mu_0$
   
   $H_a : \mu < \mu_0$
   
   $> \quad \bot$

2. Decide on the significance level, $\alpha$.

3. Find the critical value - Use t-tables instead of Normal tables.

4. Compute the value of the test statistic
   
   $$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

5. Draw conclusion.
Example:

The recommended daily dietary allowance (RDA) for zinc among males older than 50 years is 15 mg/day. The article “Nutrient Intakes and Dietary Patterns of Older Americans” reported that the zinc intake for a random sample of 25 males age 65-74 years had a mean of 11.3 and SD of 6.43.

The researcher is concerned that males in this age group may not be getting the RDA for zinc in their diets. Is there evidence to support this concern? Use \( \alpha = 0.05 \).

\[
\mu = \\
n = \\
\bar{x} = \\
\alpha = \\
s = \\
\]

1. \( H_0 : \)
   \( H_a : \)

2. \( \alpha = \)
3. Critical values? t-tables?

Large sample?

Do we know $\sigma$?

Assumption about the dist’n of zinc intake by males in the age group 65-74?

$\rightarrow$

Hence, use
4. Test statistics?

\[ t = \frac{-3.7}{1.2860} = -2.88 \]

5. Conclusion?

Reject \( H_0 \) if

Is the test statistics

Decision:

**Conclusion:**
There is evidence that the mean zinc intake by males age 65-74 is
- What type of error are we likely to commit in this test?

Type

\[ p\text{-value} = \]

(Not in tables, need computer to calculate).