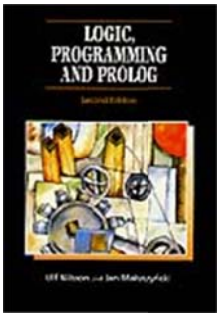
# Chapter 1 Glossary




**alphabet**: the set of *symbols* in a particular formal logic system: *constants*, *variables*, *functors*, and *predicates*. The alphabet also includes symbols for *logical connectives*, *quantifiers*, and *auxiliary characters*.

**application**: to apply a *substitution* ($\theta$) to a *variable*, *term* or *formula* (*E*) simultaneously replaces in *E* every *free* occurrence of each variable ($X_i$) found in $\theta$ with the associated term $t_i$. Variables not in $\theta$ are unaffected. See *instance*.

**arity**: the number of *terms* joined in a *functor* or *predicate*. The arity can be notated by appending a slash and the arity to the symbol: *f/n*. E.g., the relation expressed in the natural language statement, "Mary loves Tom" can be formalized as `loves(mary,tom)` where *loves* is a predicate of arity 2 (*loves/2*). Used primarily in writing; not required in Prolog syntax.

**atom**: a *formula* with no *logical connectives* (nor *quantifiers*); they are the simplest *well-formed-formulas*. Also known as atomic formulas.

**auxiliary characters**: such as parentheses ("(" and ")") and the comma(",") used to clarify the formal syntax of logic language. For example, parentheses modify binding and commas separate terms.

**bound**: a *variable X* that occurs directly after a *quantifier* or inside the subformula which follows directly after $\forall X$ or $\exists X$ is bound. If not bound, a variable is *free*.

**closed**: a formula with no *free variables*

**complete**: a set of inference rules is complete if for every set of *closed formulas P* and every closed formula *F*, whenever $P \vDash F$ (*F* is a logical consequence of *P*) it holds that $P \vdash F$ (*F* is derivable from *P*). See *soundness.*

**compound term**: a composite object constructed with *functors*

**conclusion**: a *formula* that results from applying *inference rules* to *premises*

**constants**: symbols that denote *individuals*; in Prolog, constants are alphanumeric beginning with lower-case letters. (Numbers are constants, too.)

**declarative**: in natural language, relating to statements of fact (regardless of their truth). "Mary loves Tom" is a declarative statement – even if she actually despises him.

**denote**: to identify unambiguously; to associate a literal (in-the-*world*) meaning with a signifier (*symbol*). Used to express, *e.g.,* that the *symbol* `tom` refers to a specific *individual* named Tom.

**derive**: to draw a conclusion from premises indirectly, using rules of inference

**domain**: the collection of *constants*, *predicates*, and *functors* that are the objects of a particular logic program. Also known as the *universe*. Notation: $\mathcal{D}$ or $|\Im|$

**element**: some entity in the *domain*; represented by *constants*

**existential closure**: a *formula F* of the form $\exists X_1(\ldots(\exists X_n F)\ldots)$ where $X_1,\ldots,X_n$ are *free variables* in *F*. Notation: $\exists F$

**existential quantifier**: asserts that at least one such *individual* exists. Notation: $\exists$

**first-order logic**: a formal logical system that uses *quantifiers* and includes a *domain*. *Predicates* are associated with (simple) sets. First-order logic does not allow predicates as arguments to predicates, quantification of predicates, or sets of sets – these appear in higher-order logic systems.

**formula**: finite sequences of *symbols*. See also *well-formed formulas*

**free**: a *variable* which is not *bound;* a place in an expression where substitution (i.e., *application*) may occur.

**functor**: a function over the object domain used to create composite objects. In Prolog, their symbols are alphanumeric, begin with a lower-case letter, and have *arity* > 0. Note that *foo/2* and *foo/3* are completely distinct.

**ground**: A *term* or *formula* that contains no *variables* is a ground term (or formula).

**idempotent**: generally, an operation that can be applied any number of times without changing the result. In logic, "a substitution θ is said to be idempotent iff $\theta = \theta\theta$."

**inconsistent**: a set of *formulas P* is inconsistent if both $(P \vdash F)$ and $(P \vdash \neg F)$ can be *derived*. See *unsatisfiable*.

**individuals**: unique entities in the *world* (not necessarily human beings); denoted by *constants*

**inference [rules]**: inference rules are formal re-write rules (reasoning principles) that can be used to derive new *formulas* from given ones

**instance**: a result of applying a substitution to some term or formula. E.g., $E\theta$ is an instance that results from applying $\theta$ to *E*.

**interpretation**: a *structure* plus a mapping from the *symbols* of an *alphabet* to that structure. (*Constants* map to *elements*; *functors* to *functions*; *predicates* to *relations*.) An interpretation provides a basis for assigning truth values to *formulas*. Notation: $\Im$ (black-letter/Fraktur capital "I").

**logic language**: a programming (or notation) system (such as Prolog) used to perform (or express) calculations within a formal logical system.

**logical connectives**: *symbols* used to combine *formulas* or expressions

**model**: an *interpretation* is a model of a set of *closed formulas* iff every one of the formulas is true in the interpretation.

***Modus ponens***: the *inference rule* that whenever the *premises* include both formulas $F$ and $F \supset G$, $G$ can be inferred.

**predicate logic**: Generally, any logical system wherein *variables* can be quantified, but especially *first-order logic*. The kind of logic used in Nilsson and in Prolog.

**predicate**: symbols that denote relations, written as a lowercase word followed by parentheses in which the related terms are listed. E.g., the relation expressed in the natural language statement, "Mary loves Tom" can be formalized as `loves(mary,tom)` where *loves* is a predicate of arity 2 *(loves/2)*.

**premise**: a *formula* that is given as part of the necessary condition(s) – the "if" clause – of an inference

**proof**: a sequence of *formulas* where each formula in the sequence is either a *premise* or is *derived* according to *inference rules*

**quantified**, **quantifier, quantification**: see *universal quantifier* and *existential quantifier*

**relation**: the manner in which *individuals* may be associated in the *world*. In logic programming, a relation is represented by a *predicate*.

**satisfiable**: a description of a *world* in *formulas* that has at least one *model*. See *unsatisfiable*

**semantics**: the meaning of a statement in a *logic language* as distinct from its form (*syntax*).

**sound**, **soundness**: a set of inference rules is sound if for every set of *closed formulas P* and every closed formula *F*, whenever $P \vdash F$ (*F* is derivable from *P*) it holds that $P \vDash F$ (*F* is a logical consequence of *P*). See *completeness.*

**structure**: the algebraic abstraction of the *world* – a set (of *terms*?) plus the *relations* and *functions* that can be applied to them. See also *model*.

**substitution**: a set of *variable*-to-*term* mappings: $\theta \coloneqq \{X_1/t_1, \ldots, X_n/t_n\}$. Notation: θ ("theta") or other Greek letter. See *application*

**symbol**: the syntactic representation of a *constant*, *variable*, *functor*, or *predicate*. (In a *logic language*, a symbol is encoded as a string of one or more printable characters.)

**syntax**: the precisely defined form of "legal" statements in a *logic language*

**term**: in *predicate logic*, a string of *symbols* representing some *constant*, *variable*, or *functor*; analogous to nouns (and noun phrases) in natural language. (Note that terms do not include operators.)

**universal closure**: a *formula F* of the form $\forall X_1(\ldots(\forall X_n F)\ldots)$ where $X_1,\ldots,X_n$ are *free variables* in *F*. Notation: ∀*F*

**universal quantifier**: asserts that some expression is true for all such *individuals*

**universe**: Also "universe of discourse." See *domain*. Compare to *world.*

**unsatisfiable**: a set of *closed formulas* which no *interpretation* can *model* is unsatisfiable. For example, if it includes $(F \wedge \neg F)$. See *inconsistent*.

**valuation**: a mapping from *variables* of the *alphabet* to the *domain* of an *interpretation*. Notation: φ ("phi") or other Greek letter

**variable**: a symbol that refers to some unspecified *individual*, typically notated as a single capital letter. In Prolog, the symbol is alphanumeric, beginning with a capital letter.

**well-formed formulas (wff)**: *formulas* (expressions) in a *logic language* that satisfy the rules of its *syntax*

**world**: the "reality" being modeled by the *logic program*, described as a set of *closed formulas*. Compare to *universe*