CSC/MTH447 Fall 2012
Final
Version 2.0

Take-Home Exam

Due Tuesday 12/18/12 @ noon in my Office (Tyler Rm 251)

Name: ______________________________

- Below you will find five (5) questions each worth 20 points.
- Please answer all of them.
- Please show all your work. If you get stuck on a problem at least give me an outline on the approach you were planning on taking. Partial credit is given.
- Please answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
- The work has to be your own.

1. (a) (10 points) Define a domain for the quantifiers in $\exists x \exists y[x \neq y \land \forall z[z = x \lor z = y]]$ such that this statement is true.

   (b) (10 points) Let $Q(x, y)$ be a propositional function. Prove that $\exists x \forall y Q(x, y) \rightarrow \forall y \exists x Q(x, y)$ is a tautology.

2. (a) (10 points) Prove that if A is a subset of B, then the power set of A is a subset of the power set of B.

   (b) (10 points) Assume that f is a function from A to B where A and B are finite sets. Explain why $|f(S)| \leq |S|$ for all subsets S of A.

3. (a) (10 points) Find an integer I such that $n^4 < 2^n$ whenever $n > I$. Show that your result is correct using mathematical induction.

(b) (10 points) Use mathematical induction to prove that if you draw lines in the plane you only need two colors to color the regions formed so that no two regions that have an edge in common have the same color.

4. (a) (10 points) We call a relation R **circular** if aRb and bRc imply that cRa. Prove that R is reflexive and circular if and only if it is an equivalence relation.

(b) (10 points) Assume that R is a symmetric relation on a set A. Is $\overline{R}$ also symmetric?

5. (a) (10 points) Prove that every tree can be colored using two colors. We define coloring of a tree as assigning colors to the nodes in such a way that no two adjacent nodes are of the same color.

   (b) (10 points) Prove that every tree is bipartite.