Applications of Propositional Logic

Section 1.2

Translating English Sentences

- Steps to convert an English sentence to a statement in propositional logic
  - Identify atomic propositions and represent using propositional variables.
  - Determine appropriate logical connectives
- "If I go to Harry's or to the country, I will not go shopping."
  - *p*: I go to Harry's
  - q: I go to the country.
  - *r*: I will go shopping.

If *p* or *q* then not *r*.

$$(p \vee q) \rightarrow \neg r$$

$(c \vee \neg f)$	a	$(c \vee \neg f) \rightarrow a$
T	T	T
T	F	F
F	T	T
F	F	T

Example

Problem: Translate the following sentence into propositional logic:

"You can access the Internet from campus if you are a computer science major or you are not a freshman."

Solution: Let *a*, *c*, and *f* represent respectively "You can access the internet from campus," "You are a computer science major," and "You are a freshman."

$$(c \vee \neg f) \rightarrow a$$

Note: This does not say anything about the fact when $(c \vee \neg f)$ is false, you might or might not have access.

Example

Problem: Translate the following sentence into propositional logic:

"You can access the Internet from campus only if you are a computer science major or you are not a freshman."

Solution: Let *a*, *c*, and *f* represent propositions as before, then

$$a \rightarrow (c \vee \neg f)$$

a	$(c \vee \neg f)$	$a \rightarrow (c \vee \neg f)$
T	T	T
T	F	F
F	T	T
F	F	T

System Specifications

- System and Software engineers take requirements in English and express them in a precise specification language based on logic.

  Example: Express in propositional logic:

  "The automated reply cannot be sent when the file system is full" (hint: replace 'when' with 'if')

  Solution: One possible solution: Let *r* denote "The automated reply can be sent" and *f* denote "The file system is full."

$$f \rightarrow \neg\, r$$

Consistent System Specifications

***Definition**: A list of propositions is consistent if it is possible to assign truth values to the proposition variables so that each proposition in the list is true.*

Exercise: Is this list of propositions consistent?

1. "The diagnostic message is stored in the buffer or it is retransmitted."
2. "The diagnostic message is not stored in the buffer."
3. "If the diagnostic message is stored in the buffer, then it is retransmitted."

Solution: Let p denote "The diagnostic message is stored in the buffer."
Let q denote "The diagnostic message is retransmitted",
Then the list of propositions can be written as:

$$p \vee q,\ \neg p,\ p \rightarrow q.$$

When p is false and q is true all three statements are true. So the list of propositions is consistent.

Consistent System Specifications

Exercise: Are these specifications consistent?

1. "The diagnostic message is stored in the buffer or it is retransmitted."
2. "The diagnostic message is not stored in the buffer."
3. "If the diagnostic message is stored in the buffer, then it is retransmitted."
4. "The diagnostic message is not retransmitted."

Solution: If we let p and q denote propositions as before, then the list of propositions can be written as:

$$p \vee q,\ \neg p,\ p \rightarrow q,\ \neg q.$$

Here we need to assign false to p and q in order to make the 2nd and the 4th proposition true, but this means that the 1st proposition will be false. There is no assignment to p and q which will make all four propositions true.

There are many more…

- There are many more applications of logic
  - Computer circuits
  - AI,
  - Diagnostic
  - Etc.