Rules of Inference
Predicate Logic

Section 1.6 (later parts)

Handling Quantified Statements

- Valid arguments for quantified statements are a sequence of statements. Each statement is either a premise or follows from previous statements by rules of inference which include:
  - Rules of Inference for Propositional Logic
  - Rules of Inference for Quantified Statements
- The rules of inference for quantified statements are introduced in the next several slides.

Universal Instantiation (UI)

$$\frac{\forall x P(x)}{\therefore P(c)}$$

with domain U and $c \in U$

Example:

Our domain consists of all dogs and Fido is a dog.

"All dogs are cuddly."

"Therefore, Fido is cuddly."

Universal Generalization (UG)

$$\frac{P(c)}{\therefore \forall x P(x)}$$

with $c \in U$ any element in domain U

Used often implicitly in Mathematical Proofs.

Existential Instantiation (EI)

$$\frac{\exists x P(x)}{\therefore P(c)}$$

with domain U and some $c \in U$

Example:

“There is someone who got an A in the course.”
“Let’s call her *a* and say that *a* got an A”

Existential Generalization (EG)

$$\frac{P(c)}{\therefore \exists x P(x)}$$

with $c \in U$ some element in domain U

Example:

"Michelle got an A in the class."
"Therefore, someone got an A in the class."

Our Socrates Example

$$\forall x[human(x) \rightarrow mortal(x)]$$
$$human(Socrates)$$
$$\therefore mortal(Socrates)$$

Now we show that the above reasoning step is valid, but constructing a valid argument with the same premises and conclusion:

Let U be the domain of all objects and $Socrates \in U$,

(1) $\forall x[human(x) \rightarrow mortal(x)]$ (premise)

(2) $human(Socrates) \rightarrow mortal(Socrates)$ (universal instantiation from 1)

(3) $human(Socrates)$ (premise)

(4) $mortal(Socrates)$ (modus ponens from 2 and 3)

Universal Modus Ponens

Universal Modus Ponens combines universal instantiation and modus ponens into one rule.

$$\begin{array}{l} \forall x(P(x) \rightarrow Q(x)) \\ P(a), \text{where } a \text{ is a particular} \\ \quad \text{element in the domain} \\ \hline \therefore Q(a) \end{array}$$

This rule could be used in the Socrates example.