Graphs and Graph Models

Section 10.1

Graphs

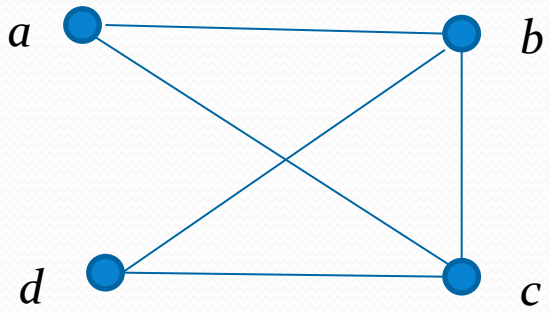
- graphs show up in many different contexts:
  - computer networks
  - social networks
  - organizations
  - tournaments
  - etc.

Graphs

Definition: A *graph* $G = (V, E)$ consists of a nonempty set V of *vertices* (or *nodes*) and a set E of *edges*. Each edge has either one or two vertices associated with it, called its *endpoints*. An edge is said to *connect* its endpoints.

Example:

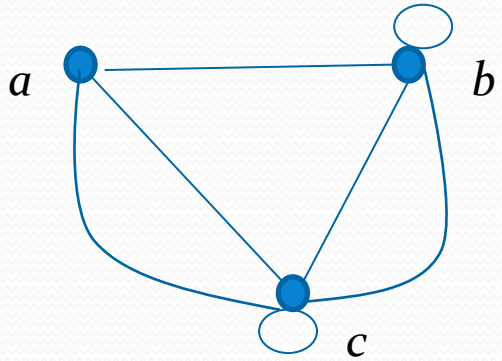
This is a graph with four vertices and five edges.



Some Terminology

- In a *simple graph* each edge connects two different vertices and no two edges connect the same pair of vertices.
- *Multigraphs* may have multiple edges connecting the same two vertices. When *m* different edges connect the vertices *u* and *v*, we say that {*u*,*v*} is an edge of *multiplicity m*.
- An edge that connects a vertex to itself is called a *loop*.
- A *pseudograph* may include loops, as well as multiple edges connecting the same pair of vertices.

Example:
This pseudograph has both multiple edges and a loop.



Directed Graphs

Definition: An *directed graph* (or *digraph*) $G = (V, E)$ consists of a nonempty set V of *vertices* (or *nodes*) and a set E of *directed edges* (or *arcs*). Each edge is associated with an ordered pair of vertices. The directed edge associated with the ordered pair (u,v) is said to *start at* u and *end at* v.

Remark:

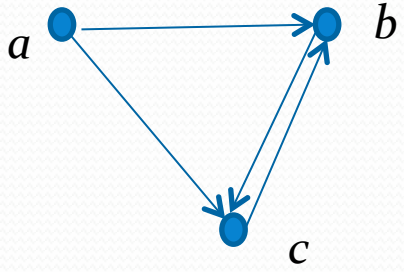
- Graphs where the end points of an edge are not ordered are said to be *undirected graphs*.

Some Terminology (*continued*)

- A *simple directed graph* has no loops and no multiple edges.

Example:

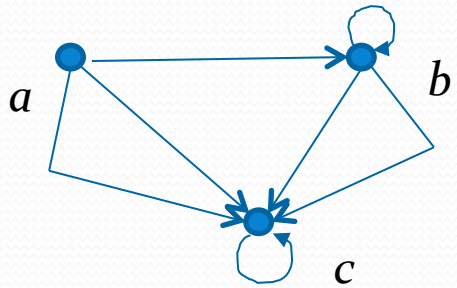
This is a directed graph with three vertices and four edges.



- A *directed multigraph* may have multiple directed edges. When there are m directed edges from the vertex u to the vertex v, we say that (u,v) is an edge of *multiplicity* m.

Example:

In this directed multigraph the multiplicity of (a,b) is 1 and the multiplicity of (b,c) is 2.



Graph Terminology: Summary

- To understand the structure of a graph and to build a graph model, we ask these questions:
  - Are the edges of the graph undirected or directed (or both)?
  - If the edges are undirected, are multiple edges present that connect the same pair of vertices? If the edges are directed, are multiple directed edges present?
  - Are loops present?

TABLE 1 Graph Terminology.

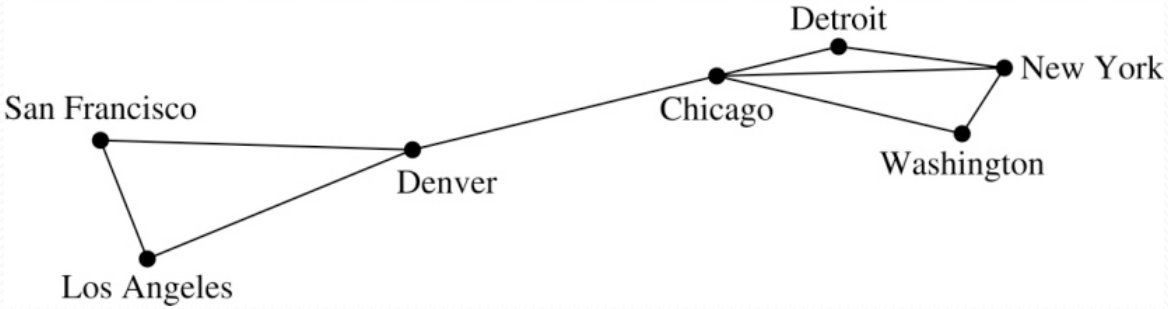
Type	*Edges*	*Multiple Edges Allowed?*	*Loops Allowed?*
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed and undirected	Yes	Yes

Graph Models

- One of the most interesting and powerful features of graphs is that they can be used to model structures.
  - We can model relationships
  - Computer networks
  - Flight schedules
  - etc.

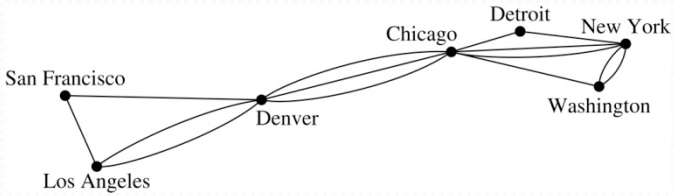
Graph Models: Computer Networks

- When we build a graph model, we use the appropriate type of graph to capture the important features of the application.
- We illustrate this process using graph models of different types of computer networks.
- In these graph models, the vertices represent data centers and the edges represent communication links:
  - To model a computer network where we are only concerned whether two data centers are connected by a communications link, we use a simple graph.
  - This is the appropriate type of graph when we only care whether two data centers are directly linked (and not how many links there may be) and all communications links work in both directions.

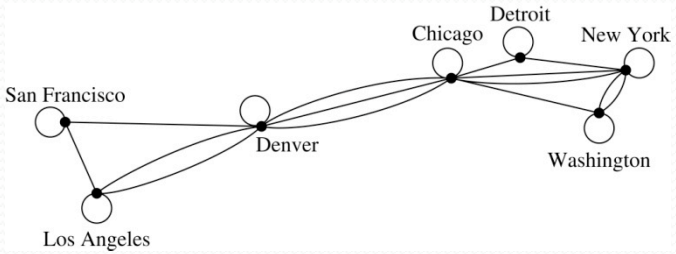


Other Models for Computer Networks

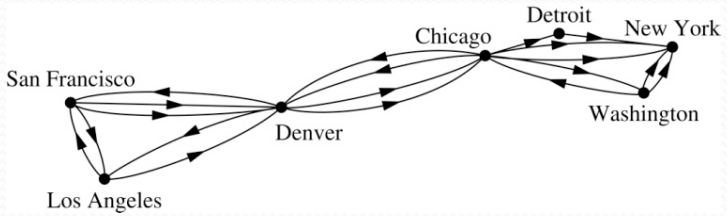
- To model a computer network where we care about the number of links between data centers, we use a multigraph.



- To model a computer network with diagnostic links at data centers, we use a pseudograph, as loops are needed.



- To model a network with multiple one-way links, we use a directed multigraph. Note that we could use a directed graph without multiple edges if we only care whether there is at least one link from a data center to another data center.

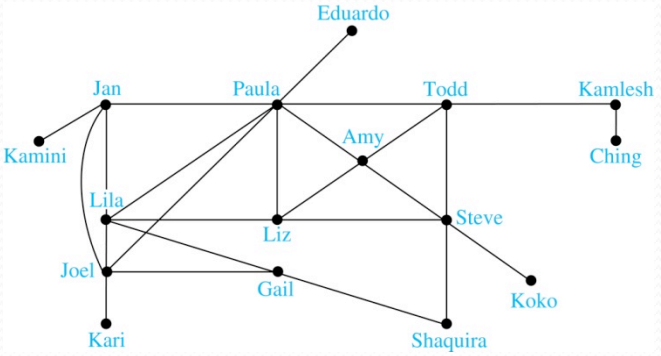


Graph Models: Social Networks

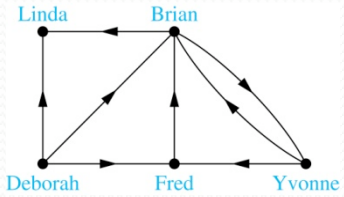
- Graphs can be used to model social structures based on different kinds of relationships between people or groups.
- In a *social network*, vertices represent individuals or organizations and edges represent relationships between them.
- Useful graph models of social networks include:
  - *friendship graphs* - undirected graphs where two people are connected if they are friends (in the real world, on Facebook, or in a particular virtual world, and so on.)
  - *collaboration graphs* - undirected graphs where two people are connected if they collaborate in a specific way
  - *influence graphs* - directed graphs where there is an edge from one person to another if the first person can influence the second person

Graph Models: Social Networks (*continued*)

Example: A friendship graph where two people are connected if they are Facebook friends.



Example: An influence graph



Examples of Collaboration Graphs

- The *Hollywood graph* models the collaboration of actors in films.
  - We represent actors by vertices and we connect two vertices if the actors they represent have appeared in the same movie.
- An *academic collaboration graph* models the collaboration of researchers who have jointly written a paper in a particular subject.
  - We represent researchers in a particular academic discipline using vertices.
  - We connect the vertices representing two researchers in this discipline if they are coauthors of a paper.

Applications to Information Networks

- Graphs can be used to model different types of networks that link different types of information.
- In a *web graph*, web pages are represented by vertices and links are represented by directed edges.
  - A web graph models the web at a particular time.
- In a *citation network*:
  - Research papers in a particular discipline are represented by vertices.
  - When a paper cites a second paper as a reference, there is an edge from the vertex representing the first paper to the vertex representing the second paper.

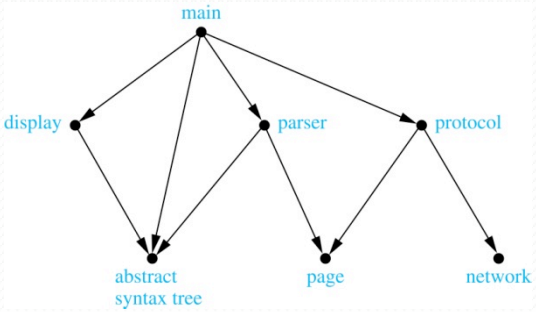
Transportation Graphs

- Graph models are extensively used in the study of transportation networks.
- Airline networks can be modeled using directed multigraphs where
  - airports are represented by vertices
  - each flight is represented by a directed edge from the vertex representing the departure airport to the vertex representing the destination airport
- Road networks can be modeled using graphs where
  - vertices represent intersections and edges represent roads.
  - undirected edges represent two-way roads and directed edges represent one-way roads.

Software Design Applications

- Graph models are extensively used in software design. We will introduce two such models here; one representing the dependency between the modules of a software application and the other representing restrictions in the execution of statements in computer programs.
- When a top-down approach is used to design software, the system is divided into modules, each performing a specific task.
- We use a *module dependency graph* to represent the dependency between these modules. These dependencies need to be understood before coding can be done.
  - In a module dependency graph vertices represent software modules and there is an edge from one module to another if the second module depends on the first.

Example: The dependencies between the seven modules in the design of a web browser are represented by this module dependency graph.



Software Design Applications (*continued*)

- We can use a directed graph called a *precedence graph* to represent which statements must have already been executed before we execute each statement.
  - Vertices represent statements in a computer program
  - There is a directed edge from a vertex to a second vertex if the second vertex cannot be executed before the first

Example: This precedence graph shows which statements must already have been executed before we can execute each of the six statements in the program.