CSC544 Spring 2015
Final

Due Monday 5/11 @ noon in my office/mailbox

NAME: ______________________

Part I: short answers (40 points).

1. (5 points) What is the the Church-Turing thesis?

2. (5 points) True or false: if L is an NP-complete language and M is polynomial-time reducible to L, that is $M \leq_p L$, then M is also an NP-complete language. Briefly explain your answer.

3. (5 points) Can $\{a^k b^k \mid 0 \leq k\}$ be considered a regular language? Briefly explain your answer.

4. (5 points) Are there deterministic polynomial time algorithms for some NP-complete problems? Briefly explain your answer.

5. (5 points) What is the difference between NP-complete and NP-hard problems?

6. (5 points) True of false: context free grammars can generate languages that Turing machines cannot recognize. Briefly explain your answer.

7. (5 points) Why can the functions Turing machines are able to compute and the primitive recursive functions <u>not</u> be considered equivalent?

8. (5 points) What do we mean when we say “NP is the class of problems whose solutions are difficult to find but easy to verify”?

Part II: long answers (60 points).

1. (10 points) Consider the *Long Way Home* problem (LWHP): here we are given a set of cities that is fully connected via roads that have different costs and the goal is to find the longest way home via a route through all the cities. Formally,

   $LWHP = \{\langle G, s, t, w\rangle \mid G$ is directed weighted graph with a maximal Hamiltonian path of weight w from s to $t\}$.

   Show that $LWHP$ is NP-hard.

2. (15 points) Show that P-space is closed under complementation.

3. (15 points) Show that A is Turing-recognizable iff A is mapping reducible to A_{TM}, that is, $A \leq_m A_{TM}$.

4. (10 points) Consider the problem of determining whether a DFA and a regular expression are equivalent. Express this problem as a language and show that it is decidable.

5. (10 points) Compute the normal forms of the following λ-expressions:

    1. $(\lambda x.\, x + 1)\,(\lambda x.\, x\, x)\,(\lambda x.\, 1)$
    2. $(\lambda x.\, x\, x)\,(\lambda y.\, y\, y)$

Extra Credit

1. (10 points) Show that the language

$$L = \{a^i b^j c^k \mid \Sigma = \{a, b, c\} \text{ and } i \geq 0, j \geq i, k \geq j\}$$

   is not contextfree.