Welcome!

- CSC445 – Models Of Computation
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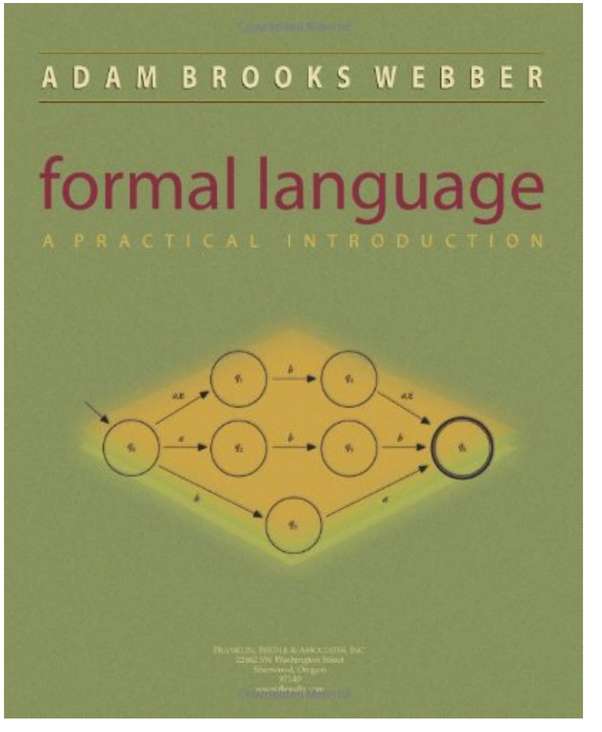

https://fbeedle.com/content/formal-language-practical-introduction

The Course

- Text: “Formal Language”, by Adam Book Webber, Franklin, Beedle & Associates, 2007.
- Special Features:
  - Course website with lecture notes
  - Online gradebook

Introduction
and
Chapter One: Fundamentals

Why Study Formal Language?

- Connected...
  - ...to many other branches of knowledge
- Rigorous...
  - ...mathematics with many open questions at the frontiers
- Useful...
  - ...with many applications in computer systems, particularly in programming languages and compilers
- Accessible...
  - ...no advanced mathematics required
- Stable...
  - ...the basics have not changed much in the last thirty years

Outline

- 1.1 Alphabets
- 1.2 Strings
- 1.3 Languages

Alphabets

- An *alphabet* is any finite set of symbols
  - {0,1}: binary alphabet
  - {0,1,2,3,4,5,6,7,8,9}: decimal alphabet
  - ASCII, Unicode: machine-text alphabets
  - Or just {a,b}: enough for many examples
  - {}: a legal but not usually interesting alphabet
- We will usually use Σ as the name of the alphabet we're considering, as in $\Sigma = \{a,b\}$

Alphabets Uninterpreted

- Informally, we often describe languages interpretively
  - "the set of even binary numbers"
- But our goal is to describe them rigorously, and that means avoiding intuitive interpretations
  - "the set of strings of 0s and 1s that end in 0"
- We don't further define what a *symbol* is, and we don't ascribe meaning to symbols

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Strings

- A *string* is a finite sequence of zero or more symbols
- Length of a string: $|abbb| = 4$
- *A string over the alphabet* Σ means *a string all of whose symbols are in* Σ
  - The set of all strings of length 2 over the alphabet $\{a,b\}$ is $\{aa, ab, ba, bb\}$

Empty String

- The empty string is written as ε
- Like "" in some programming languages
- $|\varepsilon| = 0$
- Don't confuse empty set and empty string:
  - $\{\} \neq \varepsilon$
  - $\{\} \neq \{\varepsilon\}$

Symbols And Variables

- Sometimes we will use variables that stand for strings: x = *abbb*
- In programming languages, syntax helps distinguish symbols from variables
  - `String x = "abbb";`
- In formal language, we rely on context and naming conventions to tell them apart
- We'll use the first letters, like *a*, *b*, and *c*, as symbols
- The last few, like *x*, *y*, and *z*, will be string variables

Concatenation

- The *concatenation* of two strings x and y is the string containing all the symbols of x in order, followed by all the symbols of y in order
- We show concatenation just by writing the strings next to each other
- If $x = abc$ and $y = def$, then $xy = abcdef$
- For any x, $\varepsilon x = x\varepsilon = x$

Numbers

- We use $\mathcal{N}$ to denote the set of natural numbers: $\mathcal{N} = \{0, 1, \ldots\}$

Exponents

- Exponent n concatenates a string with itself n times
  - If $x = ab$, then
    - $x^0 = \varepsilon$
    - $x^1 = x = ab$
    - $x^2 = xx = abab$, etc.
  - We use parentheses for grouping exponentiations (assuming that Σ does not contain the parentheses)
    - $(ab)^7 = ababababababab$

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Languages

- A *language* is a set of strings over some fixed alphabet
- *Not* restricted to finite sets: in fact, finite sets are not usually interesting languages
- All our alphabets are finite, and all our strings are finite, but most of the languages we're interested in are infinite

Kleene Star

- The Kleene closure of an alphabet Σ, written as Σ^*, is the language of all strings over Σ
  - $\{a\}^*$ is the set of all strings of zero or more *a*s: $\{\varepsilon, a, aa, aaa, \ldots\}$
  - $\{a,b\}^*$ is the set of all strings of zero or more symbols, each of which is either *a* or *b* $= \{\varepsilon, a, b, aa, bb, ab, ba, aaa, \ldots\}$
  - $x \in \Sigma^*$ means *x* is a string over Σ
- Unless $\Sigma = \{\}$, Σ^* is infinite
- If $\Sigma = \{\}$ then what is Σ^*?

Set Formers

- A set written with extra constraints or conditions limiting the elements of the set:

$\{x \in \{a, b\}^* \mid |x| \leq 2\} = \{\varepsilon, a, b, aa, bb, ab, ba\}$

$\{xy \mid x \in \{a, aa\} \text{ and } y \in \{b, bb\}\} = \{ab, abb, aab, aabb\}$

$\{x \in \{a, b\}^* \mid x \text{ contains one } a \text{ and two } b\text{s}\} = \{abb, bab, bba\}$

$\{a^n b^n \mid n \geq 1\} = \{ab, aabb, aaabbb, aaaabbbb, \ldots\}$

The Quest

- Using set formers to describe complex languages is challenging
- They can often be vague, ambiguous, or self-contradictory
- A big part of our quest in the study of formal language is to develop better tools for defining and classifying languages