

## Chapter 6: Index Structures for Files

index - access structure used to speed up retrieval of records  
external to the data

allows quick access to a record using a specified field as a search criterion  
- hashing from Ch 4 only permits this kind of access to key attributes

index structure - usually defined on a single field - indexing field  
- stores each value of the field along with a list of pointers to the blocks that contain records with that field value  
- values in index structure are ordered - binary search is possible  
- smaller than entire file so binary search more efficient

types of indexes:

- primary index: specified on an ordering key field  
ex: TOY db - customer file - ordered on CUST\_NUM - indexing field  
index structure contains pointer to the block containing the corresponding CUST\_NUM value
- clustering index: if several records in the file can have the same value for the ordering field
- secondary index: specified for non-ordering fields

Primary Indexes:

- two fields in the structure
- first field is of the same data type as the key ordering field of the data file
- second field is a pointer to a disk block
- one index entry for each block in the data file - total number of entries in the index file is the number of disk blocks in the ordered data file
  - non-dense index: fewer index entries than data records
- each index entry contains:
  - key value of the first record in the block pointed to by the pointer in the second field
  - first record in each block of the data file is the anchor record

(draw picture here - use TOY db)

- Problems with primary index: insertion/deletion of records
  - have to change some index entries since anchor records may change
  - possible solutions:
    - use an unordered overflow file
    - linked list of overflow records for each block
    - use deletion markers for deletion

**Example:**

Given:  $r = 30000$  records;  $B=1024$  bytes/block;  $R=100$  bytes/record

Compute:

$bfr = \text{floor}(B/R) = \text{floor}(1024/100) = 10$  records/block

blocks needed for file =  $b = \text{ceiling}(r/bfr) = \text{ceiling}(30000/10) = 3000$  blocks

binary search: #accesses approx =  $\text{ceiling}(\log_2 b) = \text{ceiling}(\log_2 3000) = 12$  block accesses

size of ordering field  $V = 9$  bytes; size of block pointer  $P = 6$  bytes; size of each index entry  $R_i = 6 + 9 = 15$  bytes

$bfr$  for index =  $\text{floor}(1024/15) = 68$  entries/block

#entries in index = 3000 (number of blocks in file)

blocks needed for index =  $b_i = \text{ceiling}(r_i/bfr_i) = \text{ceiling}(3000/68) = 45$  blocks

binary search: #accesses approx =  $\text{ceiling}(\log_2 b_i) = \text{ceiling}(\log_2 45) = 6$  block accesses

additional block access to search for record totals 7 block accesses (better than 12)

**Clustering Indexes:**

- clustering field - data file sorted on a non-key field - may not be unique
- use clustering index to speed up retrieval on such a file
- clustering index has one entry for each distinct value of the clustering field
  - index entry contains a pointer to the first block in the data file that has the corresponding field value in it

(draw picture - use TOY db - assume TOY file is sorted by manufacturer)

- insertion problem can be handled by reserving a whole block for each value of the clustering field - link together all blocks needed to store data with that value
- non-dense index because it has one entry for each unique value of the clustering field

#### Secondary Indexes:

- the first field of a secondary index is the same type as a non-ordering field of the data file
- second field is a block pointer or record pointer
- secondary index on a key field ( having distinct values for each entry in data file)
  - called secondary key
  - one index entry for each record in the data file - dense index
  - index ordered on the key field - can do binary search

(draw picture)

- with record pointers - index points directly to the location of the field
- blocks pointers - index points to the block containing the field - do linear search once block is moved to main memory
- secondary index on non-key field - 3 possible solutions:
  - 1) include several index entries with same value - one for each record in data file - dense index
  - 2) variable length records for index entries - list of pointers for each index value
  - 3) single entry for each index field value - pointer field points to a block of record pointers indicating all records in the data file containing the index field value
- secondary index usually larger than primary - so longer search
  - BUT - gain is greater since without it a linear search of the whole data file would be necessary

#### Multilevel Indexes:

- indexes of indexes - goal is to reduce the search space
- with single level indexes we have an ordered file on which we can perform a binary search
  - binary search reduces search space by a factor of 2 for each step
  - $\log_2 b$  accesses to find the desired entry ( $b = \# \text{ blocks}$ )
- multilevel indexes reduce the search space by a factor of bfr each time
  - $\log_{fo} b$  accesses -  $fo = \text{fanout} = \text{blocking factor}$
- first level of a multilevel index must be an ordered file of distinct values
- create a primary index to the first level - becomes the second level
  - one entry for each block in the first level index file
- repeat this process for the second level creating a third level index
- continue until all entries of some index fit in a single block - top level
- a multilevel index with  $r$  entries in the first level will have  $t$  levels where  $t = \text{ceiling}(\log_{fo} r)$ 
  - derive this: each level reduces the number of index entries by a factor of  $fo$
  - since we want to reduce until there is only one block we use the formula:  $1 \leq (r / (fo)^t)$

(draw picture - use from book)

#### Example:

Convert primary index of previous example into a multilevel index

Given:  $r = 30000$  records;  $B = 1024$  bytes/block;  $R = 100$  bytes/record

$bfr_i = fo = 68$  entries/block; #blocks in first level index =  $b_1 = 45$  blocks

Compute:

$$b_2 = \text{ceiling}(b_1 / f_0) = \text{ceiling}(45 / 68) = 1$$

second level is top level

$$\# \text{accesses} = 1 \text{ block at each level} + 1 \text{ data block access} = 2 + 1 = 3$$

better than 7 accesses of single level index

- Problem: insertion/deletion again
- Solution - dynamic multilevel indexes - leave space at end of each block for inserting

Dynamic Multilevel Indexes using B-Trees and B+-Trees:

Search Tree - of order p

- each node contains  $\leq p-1$  search values and  $q \leq p$  children
- at each node:  $\langle P_1, K_1, P_2, K_2, \dots, P_{q-1}, K_{q-1}, P_q \rangle$   
 where:  $P_1 \dots P_q$  are pointers to subtrees  
 $K_1 < K_2 < \dots < K_{q-1}$  are key values  
 subtree pointed to by  $P_i$  has key values between  $K_{i-1}$  and  $K_i$  (assuming values are unique)

(draw picture example)

- Problem: insertion can make tree unbalanced (all leaf nodes not at same level) - deletion can cause wasted space

B-Tree - solves these problems

- search tree with additional constraints (that solve the above problems)
- of order p
- each node contains  $\leq p-1$  search values and  $q \leq p$  children
- at each node:  
 $\langle P_1, \langle K_1, Pr_1 \rangle, P_2, \langle K_2, Pr_2 \rangle, \dots, P_{q-1}, \langle K_{q-1}, Pr_{q-1} \rangle, P_q \rangle$   
 where:  $P_i$  is a pointer to a subtree  
 $K_i$  is a key value  
 $Pr_i$  is a data pointer to record containing search key value  $K_i$   
 $K_1 < K_2 < \dots < K_{q-1}$   
 for X search key value in subtree pointed to by  $P_i$ :  
   if  $i = 1, X < K_i$   
   if  $i = q, X > K_{i-1}$   
   else  $K_{i-1} < X < K_i$   
 At most p tree pointers  
 At least  $\text{ceiling}(p/2)$  tree pointers  
   - root has at least 2 unless only node in tree  
   q-1 search key field values (data pointers)
- all leaf nodes are at the same level - null tree pointers
- insertion and deletion algs are not detailed - B+-Tree algs are because more widely used

(draw picture )

B+-Tree - most commonly used

- data pointers stored only at leaf nodes - point to block where data is located if unique (otherwise to block of pointers)
- structure of leaf nodes different from internal nodes
- leaf nodes are linked to provide sequential access to records as well as access to individual records
- leaf nodes correspond to base level of multilevel indexes - internal nodes correspond to higher level indexes
- each internal node:
  - < P1, K1, P2, K2, ... , Pq-1, Kq-1, Pq>
  - $q \leq p$
  - Pi is a tree pointer
  - $K1 < K2 < \dots < Kq-1$
  - for X search key value in subtree pointed to by Pi:
    - if  $i = 1, X \leq K1$
    - if  $i = q, X > Kq-1$
    - else ( $1 < i < q$ )  $Ki-1 < X \leq Ki$
  - At most p tree pointers
  - At least ceiling( $p/2$ ) tree pointers
    - root has at least 2 if it is internal
  - q pointers,  $q \leq p$ , q-1 search field values
- each leaf node:
  - <<K1, Pr1>, <K2, Pr2>, ... , <Kq-1, Prq-1>, Pnext>
  - $q \leq p$
  - Pri is a data pointer
  - Pnext points to next leaf node
  - $K1 < K2 < \dots < Kq-1$
  - At least floor( $p/2$ ) values
- all leaf nodes are at the same level
- pointers in internal nodes are tree pointers to blocks that are tree nodes
- pointers in leaf nodes are data pointers to the dat file records or blocks
  - except for Pnext - pointer to next leaf node
- Advantage over B-tree:
  - internal nodes do not include data pointers, so can have more entries per node - greater fanout for index files
- Search/insertion/deletion algorithms in book - look if you want

(draw picture)